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Middle School Mathematics: Intersubjectivity and the Role of Discussion in the Middle Level Math Classroom

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Plan B Project

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Abstract:

The purpose of this paper is to summarize the literature regarding Intersubjectivity (IS) and different viewpoints of how to achieve successful IS. We also examine the importance of IDE (Initiation, Demonstration, and Evaluation) sequences in the role of discussion in the middle school math classroom. In addition, this paper is a review of my Entry 3 for my National Board Certification (NBC) process; including a partial viewing of my video and written commentary used for my certification in 2014. I will examine my video and written commentary in the lens of the IS research and make connections from my entry to the research, offering examples of IS and IDE sequences from my NBC experience.

Chapter 1
Introduction

The purpose of this paper is to connect my certification experience as a National Board teacher with the current research done in the importance of group discussion. I was certified as a National Board Certified Teacher (NBCT) in 2013-2014, the same year that I started my master's program in Middle School Mathematics at the University of Wyoming. I hope to use this project to connect the research of Intersubjectivity and what I experienced during my certification process. Intersubjectivity is defined as the shared space of understanding that students arrive at through conversations and group work (Nathan et al, 2007).

The National Board standards place a high importance on discussion used in the classrooms as a tool to impact students' learning. They believe that discussions in classrooms are pivotal to students gaining a deep understanding of a concept. The main focus in two of the four entries that I had to create for my NBC was how I use discussion in my classroom as a teaching strategy. One entry focused on whole group discussion and the other entry on small group discussion. In both entries, candidates needed to show how they facilitate effective discussion in their classroom, where that effective discussion is led by the students and not the teacher.

While I know how important conversations and group work are for students' comprehension, it was beneficial for me to see the research behind them while doing this plan b project. There has recently been a push in mathematics education to use more discussion in the math classroom; that push can be seen in the published standards from Common Core, National Council of Teachers of Mathematics, and the National Board. Because the NBC process is so rigorous, I was not able to identify during the experience if the current research matches up with the newest push to communicate learning in math education. When I was able to stop and become acquainted with these ideas, I saw that communication can positively impact learning and this is not a new idea in education. In fact, some of the revered scholars in education (Piaget, Vygotsky, etcetera) have been promoting the idea of increased comprehension through discussion for a long time.

During this project, I hope to draw a connection from my third entry for my NBC and the research about the positive impact of discussions in math classrooms. While thinking about a focus for the literature review, I remembered when I was completing my entries for NBC. What were my questions or hesitations while constructing my video and written commentaries? I wanted to know if the research supported the emphasis in group work and discussion. Being raised in a traditional math classroom and teaching my first six years in a traditional math classroom, I was skeptical of the idea of conversations helping students to formulate their understanding more effectively.

<u>Research Questions:</u>

1. How does small group discussion show the evolution of students' comprehension through a conversation in a middle school math classroom?
2. How can small group discussion impact student learning in a middle school math classroom?

The review in Chapter 2 provides literature on this topic as well as insights into the questions above. In chapter 3, details about the NBC process and the partial written commentary for my third National Board entry involving small group instruction is provided. The final chapter will serve as a connection between the research and my NBC experience

Chapter 2
Literature Review

Using discussion to further understanding is not a new idea in education, Socrates the Greek philosopher popularized it in education circles almost 2500 years ago. Many scholars have researched and published information about the positive effects on students' comprehension when using discussion in a classroom. For example, Courtney B. Cazden wrote about talking during instruction and its relevance to childrens' acquisition of cognitive skills in the 1979 work *Children as Teachers- Of Peers and Ourselves* (Caszden & Steinberg, 1979). During that same time, J.T. Dillon was releasing his research on the importance of asking questions and letting students discuss their thoughts, and the role of teachers asking questions during discussion in his work *Questioning the Use of Questions*. (Dillon, 1991) Derek Edwards and Neil Mercer wrote about what conversations reveal about sharing information between teachers and peer in their 1989 work *Reconstructing Context: The Conventionalization of Classroom Knowledge*. (Edwards & Mercer, 1989) James Hiebert wrote the book *Making Sense* in 1997; in which he provided several examples of classrooms that exhibit essential features that support students' math comprehension, among the essential features listed was discussion. While using discussion in the classroom is not a new idea, Nathan et al seek to build on other's ideas, while researching a branch of discussion called Intersubjectivity and the effects of disagreements and agreements on students' understanding.

The purpose of the literature review is to provide a research basis for the National Board Certification project described for this Plan B paper. There was one article in particular that related my experience with my National Board Certification, my daily teaching experience, and the research linking the experiences to education's best practices. The main idea of the article is that understanding and encouraging Intersubjectivity (IS) is crucial to an effective learning environment. It is a foundational aspect of social interactions and cannot be avoided (Nathan, Eilam, & Kim, 2007). If a teacher wants to foster effective group learning in the classroom, one must understand IS as it relates to their students and their conversations. The authors' main point is that IS is more than the traditional view of a consensus or agreement of an answer, but IS can in fact be, "a point of convergence toward a common idea or solution" (Nathan et al, 2007). IS can happen through agreement or disagreement. By understanding IS, teachers provide understanding into students' disagreements, misunderstandings, and divergent answers; IS shows us that the flow of ideas and representations used guide students towards a converging idea and are pivotal in students' engaged discourse during problem solving. The authors videotaped a middle school math lesson and then analyzed the conversations, looking for how IS developed student's direction and understanding towards a convergent idea.

The authors (Nathan et al) sought to challenge the traditional view of IS and cited many other scholars and researchers on the progression of IS. The traditional view of IS is that understanding can only happen in a shared agreement between members. The authors challenged that view by stating that IS can occur when members disagree as well. They also stated three challenges to the traditional view of IS. First, some scholars have expressed concern over the idea that agreement is favorable and disagreement is unfavorable (Smolka, de Goes, & Pino, 1995) and research has shown that the role of disagreement in social learning is critical to the students' understanding of an idea (e.g., Piaget, 1975/1985; Vygotsky, 1978). Second, the traditional view of IS presents the argument that disagreement and agreement are different areas in a student's conceptual progression of an idea (Wertsch, 1979). Nathan et al presents that agreement and disagreement frequently coexist, but some researchers have not acknowledged their complementary nature (Matusov, 1996). Third, the traditional view of IS claims that the processes for disagreement and agreement do not move students toward successful IS, meaning that in the failed IS procedures there is no development of ideas that is seen as agreeable IS. Mortimer and Wetsch (2003) suggested that "IS should be able to address such disparate notions within the discourse. In fact, rich dialogue may actually thrive when alternative interpretations and disagreement operate within a shared context." (pg. 237-238)

Recent education reform has adopted the idea of socially mediated learning, to promote higher order thinking in all subject areas, including math (Ball, 1996; Cobb et al, 1993). Current math standards from the Common Core State Standards emphasize the use of communication as an effective learning strategy. In the Standards for Mathematical Practice #3, it is stated that students will "*Construct Viable Arguments and Critique the Reasoning of Others*" (CCSM, 2016). Before Common Core standards, the National Council of Teachers of Mathematics called for communication to be one of the five process standards considered essential to acquiring and using mathematical knowledge (NCTM, 2000).

The authors' objective in this article was to examine the structure and conversations where IS, and more specifically IS-, appeared to progress the discussion of students in the middle school math classroom, focusing on the global (changes over the entire conversation) and meso (conversations among students) levels that revealed the nature of IS. The authors used qualitative content analysis and qualitative analysis methods to focus their research on identifying the way that IS is revealed in discussion and how it shapes the social learning interactions of the students. The questions that the authors were hoping to answer through this observation are: "How does the discourse unfold over the course of the class? How does presentation use change over the discourse? What role does IS play in these changes? How is the discourse structured? What perpetuates the discourse? What role does IS play in influencing student' interpersonal interactions?" (Nathan et al, 2007)

The authors performed an observation of a middle school math classroom in which they hoped to answer their questions about the processes of IS and the role it plays in social learning. They videotaped and transcribed a lesson taught in a math classroom with 20 students. The discussion revolved around a question posed by a student to the class during a warm up: how can you get 8 equal pieces of a pie with only cutting it 3 times? The researchers went through the video tape and transcript five times, each time looking for different aspects of IS that will help them answer their questions about IS. The first viewing was to identify interactions between students; the second viewing extended the transcript by inserting information on gestures and use of representations; the third time viewing they focused on the type of representations used (environmental, graphic, or body-based resources); the fourth view used a discourse analysis perspective, focusing on students interactions with each other and how their thinking changed from that interaction; and the final viewing showed interactions that conveyed both convergent IS (students operated in a shared conceptual space, noted with IS+) and divergent IS (expressing disagreements or conflicting ideas, noted with IS-).

The authors also found that many of the interactions in the dialogue were in the form of initiation, demonstration, and evaluation/elaboration(IDE). In the past, most math classes have operated in an IRE process (a question posed by the teacher, a response by the student, and evaluation by the teacher "Yes, that's correct" or "No, that's not right"). Of the 35 stanzas, 28 were of the IDE process, and only 2 stanzas were IRE. Authors noted that 82% of the IDE sequences followed from other IDE sequences, while in contrast there were no IRE sequences connected. Researchers coded the transcript for points in the dialogue where initiation, demonstration, and evaluation or elaboration was found. All of the 28 IDE were coded IS. They found no IS codes were applied to the Initiation events and only one code to a demonstration event, and all the evaluation or elaboration segments of dialogue displayed an IS coding. Their research showed that most of the IS found in a group discussion is found during the evaluation or elaboration process, showing that IS is found mainly when students show their understanding through discussion. They also observed that 23 out of 28 IS events, were coded with both IS+ and IS- notations. When both codings were present, the discourse among students was rich and provided students the opportunity to defend their thinking and critique the reasoning of others.

In conclusion, the authors found that in most cases of IS, both disagreement and agreement were present in the same exchange. Also, the discourse structure was held together by sequences of IDE interactions, which kept the conversations moving in a productive manner. Students also used highly refined and more accessible methods of representations during the second half of discussion, while using disagreements to create dialogue (Bakhtin, 1990) and clarify their thinking and representations. This developed as students worked together and reflected on their comprehension to create a space of shared understanding with their classmates showing that perhaps the more accurate view of IS contains both agreement (IS+) and disagreements (IS-).

Students with divergent views can still interact with shared understandings and representations, as shown by Latour's *interobjectivity* model (1996). The studying of IS continues to expand as researchers seek to understand the mediated social learning dynamic (Stahl, 2006). During the videotape and examination of conversations, the authors saw students who were engaged in active problem solving and discussing their ideas in a productive, respectful, and higher order thinking environment. Socially mediated learning has become one of the markers for successful, high level thinking classrooms (Nathan & Knuth, 2003; Strom, Kemeny, Lehrer, & Forman, 2001).

Reflection

Previous research has shown that deeper understanding of mathematical concepts can be achieved through students discussing their ideas together. Communication is pivotal in students creating a conceptual understanding that they can defend while critiquing other students' reasoning. In the last several years there has been greater awareness and movement towards using group discussions in classrooms, as evidenced by the NCTM standards (2000) and the Common Core Mathematical Practices (2016). Communication between students to develop their thinking also appears frequently in National Board of Professional Teaching Standards (NBPTS), where teacher candidates prove themselves as an accomplished teacher against the National Board standards. This NBPTS process is deemed by many to be one of the highest levels of professional development for teachers.

Most teachers are aware of the research and have seen group discussion in other classrooms, but trying to cultivate an environment of group participation and communication can be daunting for teachers who have never tried it. Designing and managing socially mediated learning environments requires a different set of skills from those they have ever use before. In the past, math classrooms were often based on teacher lecture and IRE processes and there was likely little continuation of dialogue about a topic. Even problems in math textbooks, lent themselves to IDE questioning. Textbooks did not offer open ended questions or problems that permitted multiple answers, and students were not given the opportunity to defend their thinking or challenge the thinking of others. While there are still large gaps between the research of mediated social learning and educational practices (Bruer, 1997, 2006), I believe that we are starting to see a shift for the better in the way students are learning mathematics. Many curriculum creators are changing the way they write textbooks to include open-ended questions designed for group discussion. My district (Albany County School District #1, Laramie WY) just adopted a new curriculum from College Preparatory Mathematics (CPM). This curriculum is based on about 90% group work. Student complete daily lessons in class, but most are done in their small group (CPM recommends no more than 5 students per group). Questions are designed to be exploratory in nature, and teachers are encouraged to ask students to defend their reasoning and critique the reasoning of others on each part of the problem. This curricular approach is a change from the way that most math educators have taught and learned in our own math classes. Consequently, it has been difficult for some teachers to change their teaching styles to become the facilitators of learning that this curriculum requires. Teachers are now asked to guide students through learning mathematics, as opposed to lecturing and guiding students' independent practice. While the transition was difficult for me, it was gratifying to see the high level thinking that was happening in my classroom. The level of discussion among students was consistently higher than anything that I have experienced in my previous years teaching. Students were better able to relate to the mathematical principles and apply them in different ways. Going through the National Board Certification (NBC) process the year before the CPM curriculum was piloted at my district greatly helped my transition to be a learning facilitator. In two of the four entries for NBC, I had to show that I was an accomplished teacher by asking students questions to facilitate learning and guide the group's discussions back to a place that was going to lead them to correctly interpret a mathematical concept. Through this process I learned that a teacher needs to be clear in setting up expectations for group work from the first day in class. Today collaboration is a daily expectation in my class and not just a random occurrence. What I have learned while students are collaborating to make deep connections helps me ask the right questions and set expectations. I know more about what my students know and where their misconceptions lie by observing their group discussions.

It is difficult to reformat your course and restructure your classroom dynamic, but I can tell from experience that after implementing group work and discussion into my classroom, my students' comprehension has improved more than any other class. Consistent with what others have found, I have never experienced the level of engagement from my students that I am enjoying with this new style of teaching emphasizing IS (Cobb et al, 1993; French & Nathan, 2006). In my experience teaching, the top 20% of math students excel with the IRE process of teaching, but the other 80% have been left behind and forced to memorize steps to complete problems instead of understanding why they perform those steps. IS learning is allowing access to math comprehension for those other 80% of students that have struggled. I have found that the top 20% appear to prefer to do 20 problems a night over the concept that the instructor taught that day, but after a period of adjustment with this new learning style they adapt to the new format of discussion in math class. These students also like to get instant teacher feedback about the correctness of their answer. In IS, they are instead asked to defend their thinking and analyze the other students' answers, producing a more refined understanding.

A teacher's role in facilitating effective group work leading to IS is critical, so a teacher must be able to monitor several conversations at once if the class is in small groups. A teacher must also have prepped the lesson well to provide many opportunities for students to have rich dialogue. Going through the lessons beforehand is important because a teacher must be aware of many possible answers that a student might give as well as misconceptions that students might have. Converting a classroom to socially mediated learning is made easier by the use of a curriculum that promotes group work, without which teachers would be responsible for making or finding their own resources. In my experience, I have found that conducting a discussion-based classroom can be exhausting and require more prep for a teacher. Changing from a traditional classroom to a socially mediated classroom should not to be taken lightly, but the advantages far outweighed the disadvantages for my teaching.

Chapter 3
National Board Certification[1]

What is National Board Certification?

National Board for Professional Teaching Standards (NBPTS) may be considered the highest mark of professional development (NBPTS, 2016). The National Board process allows teachers the opportunity to hone their practice, show their talent in the classroom, and demonstrate their dedication to their students and their profession. NBPTS was founded with a mission to advance the quality of teaching and learning by maintaining high and rigorous standards for what accomplished teachers should know and be able to do, provide a national system for teachers who meet these standards, and advocate to reform American education capitalizing on the expertise of National Board Certified Teachers (NBPTS).

NBPTS was built on standards that were designed by teachers and the objective of National Board standards is to identify what best teachers do in their classroom every day and use it as a metric to gauge effectiveness of those seeking certification. National Board Certification (NBC) offers 25 different certificate areas, including 16 subjects and 4 developmental levels. The NPBTS was founded with five core principles of NBPTS: 1) teachers are committed to students and their learning; 2) teachers know the subjects they teach and how to teach those subjects to students; 3) teachers are responsible for managing and monitoring student learning; 4) teachers think systematically about their practice and learn from experience; and 5) teachers are members of learning communities. During every part of the NBC process,

[1] This chapter contains modified excerpts from my National Board Certification Entry #3.

candidates must show mastery of these 5 principles demonstrating that they are an accomplished teacher (NBPTS, 2016).

The National Board Certification Process

National Board Certification is a rigorous process in which candidates are required to submit 4 entries and complete 6 assessments in a testing center. In 2013-2014, I completed and was certified as a National Board Certified Teacher (NBCT) in Early Adolescent Mathematics. Interestingly, 2014 was the last year of this model for the National Board Certification process as it has since been reformatted. Assessments and entries are scored on a four point rubric, with a total of 400 points possible. In order to certify as a NBCT, candidates must score a 275 (NBPTS, 2013).

Entry one was designed to evaluate the candidate's ability to differentiate instruction. In this entry, candidates show their ability to evaluate learning strengths and meet the needs of individual students. Candidates must plan and implement appropriate differentiated instruction, as well as analyze and modify instructional strategies and materials based on ongoing assessment. To complete this entry, candidates must choose two instructional activities from two different students that demonstrate how candidates are able to design a sequence of learning experiences that builds on and gives them insight into those students' conceptual understanding of a mathematical idea. Through a written commentary, candidates reflect, analyze, and describe their teaching, as well as show how their instructional decisions impacted students' learning (NBPTS, 2016).

Entry two shows how an accomplished teacher can use targeted questioning and class discussion to explore an important mathematical topic. Discussion must be in a whole class format and an accomplished teacher must show how to engage students in discussion as the whole class explores, investigates, and discovers the math concept. Teachers must show evidence of providing an environment that promotes student learning of mathematical procedures, concepts, or reasoning processes. The candidate must submit a 15 minute video and written commentary that shows evidence of these standards and five core principles (NBPTS, 2013).

Entry three shows a lot of the same standards as entry two but discussion must be in the small group format. There is also an emphasis on using manipulative or technology to provide access to and deepen mathematical thinking. An accomplished teacher must show ability to use questioning strategies and mathematical thinking and reasoning processes that promote mathematical discourse between the students. Candidates must again submit a 15 minute video and written commentary that shows how the teacher met the National Board standards and five core principles (NBPTS, 2013).

During the final entry, candidates are asked to write about their accomplishments in teaching and how those accomplishments impacted student learning. Candidates must also demonstrate their involvement in learning communities that positively impact student learning. This entry is designed to show the relationships between accomplished teachers, their students, their community, and other professionals. Candidates are required to submit documentation and a written commentary showing how the candidate has met the National Board standards relating to this entry (NBPTS, 2016).

The assessments for the NBC consisted of six 30 minute tests administered at a National testing center. Topics for a candidate certifying in Early Adolescent Mathematics included: Context for Mathematics, which included historical development of mathematical ideas, math application in fields related to math, precise communication of mathematical ideas; problem solving and number sense, including numbers and operations, algebra and functions, and geometry; and finally modeling and analysis, which consisted of trigonometry, discrete mathematics, data analysis and statistics, and calculus (NBPTS, 2016).

NBPTS Entry Three

I have chosen to include my video tape and written commentary (as per NBPTS regulations, only part of the process can be shared) from my Entry Three; these helped me to answer my research questions based on my teaching and certification experience. My entry first showed me the evolution of students' conceptual understanding from basic to proficient using small group discussion. Because I completed my National Board Certification first, I developed my research questions to view from a research based perspective what I observed during this process. I have first included the instructional context that gives the reader a snapshot of my students and my teaching during this unit. The planning section displays how I prepared to teach the concept of variation to these students. The third section is an analysis of the video recording. It describes the lesson, what is observed in the video, and the conversations that happened during this lesson. The final section is a reflection of the video in regards to the National Board Standards.

Instructional Context

This Algebra 1 class consisted of 19 Students, 13-14 years old and the unit covered pertained to Direct and Inverse Variation. Algebra 1 is generally a 9th grade course at our school, but all of my students in this class are 8th graders. These students were recommended by previous math teachers and their test scores qualified them to be in an advanced math class. This means I cover the same material as other Algebra 1 class, but I can engage them in a deep discovery of topics. My class is more advanced than other Algebra classes and units go quicker, leaving us more time to apply topics to real-world situations. I have a few of the outspoken students that will take over the discussion in a big group, but small groups or pairs work well for discussions and activities because they all get the chance to contribute to the conversation.

When putting student into groups, I have students physically change the environment of the room to promote effective group work and discussion. We move desks in the groups to face each other, setting the environment for both the non-verbal and verbal engagement from the beginning. This also allows students to feel comfortable dialoguing and asking questions with each other. This class has proficient math skills and is most comfortable learning through verbal instruction followed by examples.

My students know that direct variation is a relationship between numbers, usually positive numbers, that when graphed appear as a line passing through the origin. Prior to this unit, we investigated graphing lines and slope-intercept form. Students are familiar with linear equations and know how to write/graph them. Inverse variation is a new type of function that these students have not seen before this unit, so students often benefit from real world applications to help them think and reason mathematically about this concept. When comparing variations, these students may benefit by seeing real-world application and kinesthetically experiencing the topic.

Variation is a great math concept to incorporate the use of graphing calculators, so I have introduced and used graphing calculators every lesson during this unit to allow students to see connections graphically and numerically through the list function.

Planning

The daily learning goal for the video-taped lesson was "Students will compare data, graphs, equations, and charts and make a hypothesis as to which variation each scenario represents." The unit objective was "Students will be able to use and apply rate of change and variation." This year I found the activity in our textbook and modified a few of the questions in an attempt to make the activity activate deeper understanding of variation and build on students' interests. The four class periods before this lesson, we covered direct and inverse variation. We looked at how the equation is formed, what the k (constant of variation) is, what the graphs look like, and what the data table looks like. Students have developed a strong mathematical foundation in what variation is and how we identify it through data and graphs. Through informal and formal assessments (checks for understanding during instruction, short daily quizzes over their homework, and checking their homework), I was sure that students had achieved the necessary level of understanding to be successful applying their foundational knowledge to this activity.

This class is capable of discussing ideas in small groups, but they get frustrated or upset when I do not directly tell them an answer. They all have had success in their previous math classes because math came easy for them, but I want them to develop the character to persevere when things get hard so I will often make them talk out their questions together to get them to use reason and logic to answer their questions. I would like them to be comfortable and experienced working together and have the perseverance to solve problems together. I allowed students to pick their group for this activity, because I hoped that this opportunity would enable students to be highly active during discussion and thus highly likely to contribute to a group in which they feel comfortable.

Using graphing calculators provided the opportunity for students to relate data and graphs to technology. The graphing calculators provided instant feedback to the students about their variation hypothesis. Students were able to relate the table and the graph to what they know variation should look like, supporting or disproving their hypothesis. When students used the technology correctly, they were able to access deeper levels of understanding, while still relating the data into real world situations. I brought in a multiple speed bicycle the day before so students could gather the data needed to make the variation equations and tables, and they also used the bike to test their hypothesized equation after their discussion in this lesson.

Analysis of Video Recording

We started this lesson by reviewing our group work and discussion expectations that we established the day before and we also reviewed our daily objective and I told them how I would know if they achieved that objective (written answers on the worksheet that accompanied this activity, the daily quiz tomorrow, as well as conversations that happened during discussion time). After the video recording, students continued to use their equation to make their hypothesis on number of wheel revolutions in their chosen gear and then performed the experiment on the bike to see if their hypothesis was correct. We concluded by students handing in their worksheets so I could check for understanding that night. Subsequently, I gave a short quiz the following day to see if students could use their understanding about variation given different numbers as it applies to bicycle gears. We continued to reference this activity throughout the rest of the unit as this understanding helped students to draw connections to newer content and prepare for the test.

I know that students achieved the learning objective by checking their homework that night, paying special attention to their answers to the following prompts: “describe how the number of teeth affect the turning of the wheel”, “what kind of variation is this”, and “write an equation that relates the number of teeth to the number of wheel revolutions, what is the meaning of the constant in this equation?” Between the answers on the worksheet and their daily quiz (they averaged a 4.1/5 or 82%), they proved that they knew and were able to apply their knowledge of variation; they also averaged an 86% on their unit test.

Evidence of students achieved the learning goal was shown through several comments and interactions on the video: Black/Pink’s excitement over her making connections- arms up, yelling “Yes!”,”Wait! I get it!”; the groups excited comments “bam!”, “it works!”, “Yay!” when they matched their equation to their data points; Black/Pink comment “Find k first cause the equation is y=k something x. We have to find if it is add, subtract, multiple, or divide”; the girl in the Green nodding in agreement with Black/Pink mathematical reasoning “This is y and this is x, because this divided by this is this.”

I wanted the black/pink’s group to know another way to check their equation besides the graph, so I used questioning strategies that prompted them to check the other data with their equation to make sure it worked for all the data. During discussion the girl in the Black/Pink kept trying to engage me with questions, I responded by acknowledging her question, but directing her to ask her group. When she was struggling to find the constant, I used questioning strategies and promoted mathematical discourse by encouraging her to ask her group, the girl in the cream shirt said “Wouldn’t it be L1/L2? The constant would be the number of teeth on the rear sprocket." This allowed the group to communicate their mathematical reasoning and use discussion to make the connection, demonstrating fairness and access to learning for their whole group. I provided guided organization for their thoughts to the black/pink's group, to help them make connections; they needed a little more help than the other groups to organize their thoughts and communicate together. They also needed more encouragement to keep thinking through their questions, as evidenced by the following interaction: Black/Pink: "This is confusing!" Me: "Keep going, you are on the right track. Anna (Cream shirt) what do you think?" Green shirt: "I did this and it worked." Me (modeling mathematical thinking and reasoning): "Ok, so this divided by this gave you this. What does 30 represent? Y,K, or X?...Maybe you should mark above the column which data is what letter?" Without the direction to organize their thoughts, they were starting to talk in circles, confuse themselves, and were not able to understand what their partner was trying to explain, as evidenced by this exchange from the girl in the black/pink. She was very excited about the revelation that 30/28 is 1.07 and that this was consistent through her data, but the girl in green had made that revelation a few minutes earlier and tried to communicate that to her group. For some reason, they had missed making that connection together. By stepping in, I was able to focus their conversation back to a place where it was going to start being productive again, and they could begin to build connections together. At the beginning of the video, I was circulating and heard the start of this group's conversation. I asked the videographer to stay with the group because in my experience teaching this topic, I suspected this conversation was going to demonstrate a great evolution showing the group's mathematical thinking and reasoning.

When students in my class reach the appropriate level of understanding, they know that it was their responsibility to make sure their group reachs the same level; as evidenced by the video when the girl in the Black/Pink Shirt got her equation to match her data points. I asked her to help her group and make sure they got the same outcome on their calculators. There were several interactions on the video that showed students' thinking and reasoning mathematically and communicating it with each other. The first one we see is the girl in the grey/navy who said, "Yeah that's not right cause its curved" to which the boy in the grey/green replied "Shouldn't it be straight?" They knew that their data showed a direct variation, but their equation on their graph was showing an inverse variation (curve). During the video you see me model mathematical reasoning by asking the black/pink's group, "So if you had the right equation, it would connect your data?"

The calculator was a great tool with this activity to help students establish their hypothesis and check their understanding. During the video, this hypothesis establishing was captured by the girl in the cream when she said, "Couldn't it be either equation?" Black/Pink replied "Well let's see." She checked the hypothesis by graphing the equation in her calculator. The girl in the cream shirt was wondering if their data could be both equations they were discussing and the girl in the black/pink shirt used the technology to graph them and see if this was possible. The calculator provided a visual picture for their mathematical thinking and reasoning. Using technology, most students were able to make the correct connections to make this learning activity academically successful. The bike provided an opportunity to get students to apply the equation that they developed back to the real world situation. This served as another built in opportunity for students to interact with the data and test their hypothesis.

Reflection

I think one of the main successes of this lesson was the dialogue that happened within most of the groups. The black/pink, boys, and the pink shirt's groups had some high level questioning and real world connections being made through thinking and reasoning mathematically, and they communicated it well. When these students were allowed the opportunity to make connections as a group through exploring and discussion, the connections they made were stronger than usual and their understanding became well-rounded.

Working in small groups can be hard to manage as a teacher. With this lesson I should have assigned groups to get the best group chemistry and to allow the discussion to be as productive as it could be. I would have pulled grey/navy girl out of her group and put her in a pairing with one of the girls from the girl group. I learned a lot about her personality through this activity because I have seen her work in pairs this year and she does well working with one other person. This way she would not be able to dominate a whole group and would be forced to collaborate with only one other person. Her original group tried to engage her in discussion, but her personality does not enable her to handle whole group or 4-5 member small group interactions well. For example, she took the calculator out of Pink's hands and did the computation for her. At that point, Red asks, "Wait how did you do that?" The boy in the Grey/green tries to engage the group in a question, but Grey/Navy and Pink keep talking over him. Red (not in frame) tried to answer his question, but discussion was not productive or conducive to learning. While I saw great conceptual understanding come from the discussion in small groups, it did reinforce the idea that small groups need to be purposeful in set up and activity design. All the groups' discussion must be monitored to ensure that they are on task and learning is happening. Moreover, the class must also have a set of group work expectations that are known and enforced.

Chapter 4
Tying it all together

Overall, my small group activity for my National Board Certification Entry 3 was successful in regards to student comprehension. My NBC experience helped develop me into a teacher that is better able to implement group work in my class, especially since our district would adopt a curriculum built largely on small group work in 2015-2016. I found the transition smooth for my teaching style because of this NBC experience, while some of my peers struggled with adapting their teaching styles to the new curriculum. I was also able to commit fully to the curriculum, because I had witnessed the increase in student comprehension through small group learning and discussion.

Unfortunately, because of NBPTS regulations the video cannot be viewed by non-NBPTS individuals so the reader cannot witness what I am describing in my video. If you were able to see my video, you would see my marked examples of IDE sequences and IS+ and IS- interactions between groups. You would also see the progression of a specific group through many IDE sequences and finally landing in that shared space of understanding (Intersubjectivity), which is also the same moment that the learning goal was achieved. It is also worth noting that the emphasis was not on precise/accurate mathematical language for this lesson, that emphasis will come later in the chapter.

Classroom Environment/Atmosphere

Effective group discussion will not happen accidently and must be carefully implemented by the teacher. In order to promote a classroom that has effective group work, a teacher must carefully set up an atmosphere and clarify expectations when setting up their classroom at the beginning of the year. The teacher must first define what effective group work is and how it looks and sounds in their class. While I was not familiar with the term of Intersubjectivity before my research, I did know that I wanted my students to eventually come to shared space of understanding. I figured out after a few months that students could arrive in this space while in disagreement, as long as this was a productive and respectful conversation. Teachers who wish to use discussion in their classroom must first discuss with their students what a constructive discussion must look like and decide how they will hold them accountable to that expectation. Intersubjectivity cannot be used well in the context of group work unless the students have accepted the role of discussion in their learning. I have found while setting up expectations in the first week of school that students benefit from knowing why the classroom is designed this way that it is. Sharing the research of IS and the importance of discussion in their learning allows students to accept the new way of math class.

I have also found that changing the physical environment helps students to start and continue the discussion with their teammates. If students are not encouraged to use discussion in their group, they will often resort to working alone. Because teachers understand the importance of discussion for the student's comprehension, they need to enforce the expectations of discussion in class as well as monitor the groups' discussions. The first few weeks of school are critical to formulating a class environment where discussion can thrive. Working together and discussing a topic is a vulnerable thing for teenagers to do, but we must encourage them to share their thoughts and make the environment a productive place for IS to succeed.

It is important to share expectations with students for what their conversation should look like, but also what the end product of their discussion should sound like. In most cases, I want my students to end a discussion in Intersubjectivity, or a shared space of understanding. This concept is easy for students to understand. In fact it tends to be harder for them if they are allowed to disagree at the end of a conversation than if they had to reach an agreement to conclude the discussion. Mathematics students have usually experienced a situation in which there is a right or wrong answer. Open ended questions in which there might be multiple answers or no correct answers, tend to make them uncomfortable. Most of my preparation with students in group work settings is teaching them how to operate if there are disagreements (IS-). Students are usually respectful to one another. While students can easily learn to express their thinking, it is much more difficult to think through a divergent idea expressed by a teammate. Students have to first understand what their teammate is expressing, evaluate their thinking to see if they might be correct, and then restate their original position in light of their teammates' stance. This process is difficult for students to do in their early adolescence. However it deepens their understanding of that concept and teaches them how to communicate their reasoning.

The class featured in my Entry Three is the top in math comprehension of their class. In Chapter 2, I spoke about how these students have succeeded in the traditional classroom and lecture format of math class. There was a lot of preparation work that allowed this class to have a discussion with that degree of productivity in the video. We had done group work several times a week since the first week of school. I first needed to train them to work without being led by a teacher. Then I had to teach them how to have constructive, but respectful conversations, especially if someone had an alternative view or answer. Perhaps the most intensive training that we experienced was pushing them to evaluate and critique the reasoning of others. We practiced all of these skills in group situations. As a teacher I had to continually monitor their conversations to make sure they were following our agreed upon rules for discussion. When teaching students in a different style, the teacher must be patient and unyielding with what they expect their students to do. We saw some great conversations develop in that class as was shown in the video, but that discussion did not happen by chance. The students and I have worked hard to interact in a constructive manner, and the high level thinking that took place was completely worth the effort.

Final Observations

I see in the video and written commentary for my Entry Three many examples of Intersubjectivity. The video showed a great progression of a group (Black/Pink girls) working through understanding, finally landing in a shared space of understanding. During their progression, they had to work together and rely on one another to keep their momentum and conversation moving in a productive manner. In many cases, it was the girl in the Black/Pink that was talking and moving their discussion in a positive way, but without her teammates, the team would not have arrived at the correct conclusion.

We did observe along the way that IS can be inhibited by poor communication or not listening to one another in the group. In the video, the girl in black/pink was so excited to find the constant of variation, but one of her teammates had already made that conclusion a few minutes earlier in the discussion. Although IS was eventually achieved in that group, it could have been more efficient and productive if not for the poor listening skills of the girl that worked in a mostly verbal manner.

In this situation, the graphing calculator technology provided students a great opportunity to achieve IS. The use of manipulatives or technology can assist students when trying to defend their reasoning and critique the reasoning of others, as it was a powerful tool for students in this situation.

In regards to IRE (initiation, response, and evaluation/elaboration) and IDE (initiation, demonstration, and evaluation/elaboration) sequences, there were two great examples of an IDE sequences in the video, while there were no IRE sequences shown in the video. IRE sequences are common in most classrooms (Lemke, 1990) and there are times in math class where a student needs direct feedback about an answer, but when a teacher is trying to develop a student-led discussion, IRE sequences are not helpful. The two great examples of IDE sequences seen in the video, both came from the black/pink's group during the process of finding the constant and writing the equation. The first initiation came from the worksheet questions, the student in the black/pink answered in a demonstration, and her teammates gave her an evaluation and elaboration. A question came out of this evaluation and elaboration that led them directly into another IDE sequence. What was most evident in relationship to sequences was how one IDE sequence led into another. Almost the whole video is IDE sequences linked together. There were times that I had to come in and provide an initiation for the black/pink group because their conversation had stalled or they were not organized enough to make the necessary connections. There were several situations in the video where I was answering questions from students without turning them back to their group, but in these questions I was problem solving technology issues with that student.

I do not think that I saw a true example of IS- during the video segment. When the groups were discussing their thoughts and trying to figure out an answer it can look like IS-, but the members did not come from divergent thoughts to a common shared space. I saw groups discussing their ideas, but those ideas were not fully formed in that they could defend these ideas. I probably could have seen more IS- situations if I had let the students work by themselves on the worksheet first, then come together as a group to discuss their answers. However, I needed to show evidence of the progression of understanding in a small group discussion to meet the National Board standards for Entry Three. Once a group had a minor IS or shared understanding together, they tended to operate in IS+ (Students coming from agreeable views to achieve a shared space of understanding) space for the majority of the rest of the time, this was seen nicely in the Black/pink's group.

When the video showed the group with the grey/navy girl and their conversation, it was a dysfunctional group that did not collaborate or listen to one another. There were times that one person was trying to explain their reasoning, but the group was not giving that person their attention. Therefore, their group did not reach an IS position in the video; they did eventually reach the level of understanding needed to achieve the daily objective, but it was as I guided them through the discussion.

While watching the video in the viewpoint of this research, I was struck by how easy it would have been for me to switch from the IDE questioning to the IRE questioning. For teachers trying to implement discussion in their classroom, they must be disciplined in how they chose to respond to their students' questions. Because a teacher's instinct is to answer their student's questions, it was difficult for me to turn the question back to the group. However, I knew that if

I was going to have a productive environment for discussion, I needed to do my best to let the students critique the reasoning of others.

Overall, I was amazed at how often I observed the Intersubjectivity and IDE sequences in my Entry 3 video and throughout my NBC experience. It seems that best practices for group work discussion naturally encourages IS and IDE sequences. IS seemed to be a natural progression if the teacher is dedicated to letting their students learn through discussion, critique the reasoning of others, and defend their thinking. Having gone through the experience of my NBC and after reading the research about IS, I now realize how important discussion is for my students' understanding. One of the biggest things that I learned through the research of IS, is the fact that IS can be obtained through differing viewpoints. It is a good reminder for me as I monitor group discussion that just because a group is not in agreement at the moment, that IS can still be achieved at the end of the conversation. The other big takeaway for me from the research was the importance of IDE sequences in the progression of a groups' discussion and even when lecturing during class. While I continue to seek to improve my teaching for students, I know that I will implement the research done in this project as well as the experience from being a National Board Certified Teacher every day.

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