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Learning Mathematics the Write Way: How does Writing in Mathematics affect Mathematics Learning in Middle School?

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By

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Abstract

Writing in mathematics has been recommended by professionals and researchers for almost 30 years. The National Council of Teachers of Mathematics, Common Core State Standards, university professors, and employers have emphasized the need for individuals to write in mathematics. The purpose of this literature review is to explore and possibly make recommendations about using writing to help solve mathematics problems in middle school mathematics classes. This paper examines what the literature tells us about the use of writing in solving mathematics problems, what ways writing in mathematics can affect middle school comprehension, and how writing in mathematics can affect students' metacognition. I also discuss how my findings in the literature apply to my classroom practice as a middle school mathematics teacher. Writing in mathematics has the potential to give the teacher valuable information, express students' mathematical understandings, help students see mathematics as a process, and connect students' mathematical ideas. In general, students tend to have greater mathematical ability in classrooms where mathematical writing is used. The effectiveness of writing in the middle school mathematics classroom depends on the classroom environment, scaffolding, frequency of student practice, and purpose of the writing assignment.

This thesis is dedicated to my partner and husband, Ian Abernethy, who has been a boundless source of support and who has helped me be the best middle school teacher I can be. It is also dedicated to our first child who has been developing in utero with the progression of this thesis. Finally, I dedicate this thesis to our two old English sheepdogs, Fezziwig and Penelope, for bringing out the best in me and always making me feel loved.

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Chapter One: Introduction

Background and Problem

Writing in the mathematics classroom has been recommended by researchers and professionals for almost 30 years. In 1989, the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* highlighted mathematical communication as one of five major goals for deeper understanding (NCTM, 1989). In 2000, the National Council of Mathematics Teachers again emphasized communication in the mathematics classroom in its *Principles and Standards* (NCTM, 2000). The Common Core State Standards in Mathematics (CCSS-M) supported the NCTM's initiatives by maintaining students reason mathematically, construct viable arguments, and attend to precision. The CCSS-M standards include communicating precisely, using clear definitions, stating the meaning of symbols, and including units (Martin, 2015). Communications in these standards include the use of speaking and writing so students can provide rationale for their mathematical thought processes.

I've used writing in my three middle school (5^{th}, 6^{th}, and 7^{th} grade) mathematics classes for over five years. Throughout this time, I have developed several ways for students to write in mathematics class:

- Assigning students to write a paragraph explaining how to solve a specific mathematics problem (e.g., what is 4/5 of 60) or to write a paragraph explaining a particular mathematics concept (e.g., explain how to multiply fractions);
- Having students write a few sentences in their notebook at the end of class to answer the day's essential question (e.g., Essential Question: How do you

calculate the area of a circle? Answer: Calculate the area of a circle by multiplying the radius squared by pi. Remember to square the radius first and include square units.)

- Having students get in a big circle to answer the day's essential question, with each student contributing one word as a scribe writes on the board, followed by editing and students writing it in their notebooks;
- A "silent conversation" where students write about a mathematics concept on a small paper and pass their paper to the next student for them to add to the "silent conversation," and this continues for several students;
- Having students answer story problems in a complete sentence;
- Having students always include correct units when necessary; and
- Having students write their own story problems about a particular mathematics concept.

The first writing activity I used as a first-year teacher was asking students to write a paragraph about solving a mathematics problem. Writing paragraphs in mathematics class was my principal's idea, which I readily employed on a weekly basis. Starting in second grade, students at my school should be able to write a paragraph. They learn about topic sentences, supporting details, and concluding sentences. Writing conventions are reinforced and expanded upon in the third and fourth grades. However, by the time they get to fifth grade and are asked to write a mathematics paragraph, they often forget things as simple as capital letters, periods, verbs, and writing coherent sentences. When no scaffolding is provided, I get a wide range of paragraphs: a paper with just numbers and symbols and no words, mathematics steps in the middle of a paragraph with words around it, a series of incomplete sentences, or a coherent and

informative paragraph. Since many students struggle with clarifying their mathematical thinking in writing, Stonewater (2000) recommends teachers provide a framework for helping students learn to write in mathematics.

When I started the process of assigning mathematics paragraphs, my principal said scaffolding was a good idea because you cannot just expect middle school students to know how to write a mathematics paragraph. My principal's advice prompted me to provide a lot of scaffolding at the beginning of the year. For example, I provide a fill-in-the-blank for the first few weeks of school (Fig. 1).

Written Practice question _____on page _____ asks us to solve_____. This problem is about _____, which we learned about in Lesson _____. To solve this problem, _____because_____. The answer to the question is _____. To check your answer _____.

Figure 1. Fill in the blank for students to explain how to solve a mathematics problem at the beginning of the year. I use this when students first learn to write a mathematics paragraph explaining how to solve a specific mathematics problem. The blanks are not proportional to how much information students should write.

After one to two months, students transition from the fill-in-the-blank to a list of steps (Fig. 2). As the year progresses, I provide less scaffolding. Regularly, I remind students to use vocabulary in their writing and explain why they implement the necessary steps. By the end of the year, I aim for students to view writing in mathematics class as part of their normal routine.

Step 1: State the question:
"Written Practice question 12 asks _____."
Step 2: Explain the concepts that pertain to the question:
"This question is about_____. We learned about this in Lesson _____."
Step 3: State how you'd solve the question, include why you would solve it this way:
"Solve (rewrite the question here) by _____. You would do this because_____."
Step 4: State the answer:
"The answer to the question is _____."
Step 5: State how would you check your answer:
"Check your answer by_____."

Figure 2. Steps to writing a mathematics paragraph to be used with middle school students after they have mastered the fill-in-the-blank. The blanks in each step are not proportional to how much information students should write. They can expand on their ideas if they think it is necessary. Students should keep these steps and refer to them later in the year.

After students leave my middle school mathematics class (and all middle school mathematics classes), they eventually go on to high school, and later go on to the university and/or the workplace. Few students arrive at the university level having been required to write about mathematics in secondary school (Pugalee, 2004); and Madison (2012) implies mathematics majors cannot write. When university students are asked to write in mathematics classes, their writing is often unclear and inaccurate (Van Dyke, 2015; Stonewater, 2000). Knowing how to write in mathematics can be helpful both in the workplace and to improve public discourse (Madison, 2012)**.** Kennedy et al., (2014) claim writing can serve a critical role across all disciplines, and it can be used to build connections between mathematics and other subjects.

Countries such as South Africa, Australia, England, and Wales use writing to promote deeper understanding in mathematics classrooms (Ntenza, 2006). At the international level, writing in the mathematics classroom has been in place since the time of the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (Ntenza, 2006). More writing at

all levels of mathematics has been recommended in the United States (Van Dyke et al., 2015), but despite these recommendations, it has not gained wide acceptance in the mathematics classroom (Pugalee, 2004). Writing in mathematics is considered to be an area of investigation because it provides opportunities for students to show mathematical understandings and shape mathematical arguments (Martin, 2015). Braun (2014) states that if we "want to change outcomes for our students…we must reconsider what our students are doing and what they are being asked to do" (p. 449). Many students in the United States think the main point of mathematics class is getting the right answer (Van Dyke et al., 2014).

Purpose and Questions

I am studying how middle school students write about solving mathematics problems because I want to find out how this affects their learning. The purpose of this literature review is to explore and possibly make recommendations about using writing to help solve mathematics problems in middle school mathematics classes. This is to help my reader understand if and how they can use writing in mathematics to synchronously improve students' writing and mathematics comprehension. This literature review is guided by the following research questions:

Questions

What does the literature tell us about the use of writing about solving mathematics problems in mathematics?

- In what ways can writing about mathematics affect middle school students' mathematics comprehension?

- What does the literature tell us about how the use of writing in mathematics affects students' metacognition?

These questions are important to me because I have used writing in the mathematics classroom for over five years, and after reflecting on my classroom experiences, I find writing is beneficial to students. I would like to see if the literature supports the idea that writing in mathematics is beneficial.

Methodology

This literature review involved review and synthesis of peer-reviewed journal articles pertaining to the topic of writing in mathematics, especially as it relates to middle school students. The purpose of this literature review is: (a) to find answers to my research questions and (b) to explore how my teaching practices relate to my findings in the literature.

To find pertinent publications, I conducted key word searches using databases like, ERIC, PsycINFO, and Google Scholar. Examples of key words I used to search the databases include *writing in mathematics, mathematics writing, mathematics writing in middle school, mathematics writing and metacognition, history of writing in mathematics, mathematics discourse, mathematics discourse in middle school, how does writing affect mathematics*, and others. After reviewing the literature, I synthesized the information. Occasionally throughout this process, I went back and obtained and reviewed additional peer-reviewed articles and added this information to my synthesis.

Chapter Two: Literature Review

History of Writing in Mathematics

In 1989, mathematics communication was identified as one of five major goals by the National Council of Teachers of Mathematics (NCTM) in their *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). This involves elementary school, middle school, and high school students reflecting on and clarifying their own mathematical thinking, discussing mathematical ideas, and making mathematical arguments (NCTM, 1989). With this publication, the NCTM suggested mathematics teachers create an environment where students practice and refine their growing ability to communicate their mathematical thoughts both verbally and in writing (NCTM, 1989).

In 2000, the NCTM again emphasized mathematics teachers create an environment to foster writing in the mathematics classroom in its *Principles and Standards* (NCTM, 2000). This document emphasized writing in mathematics classrooms so that students could test their understandings on the basis of shared knowledge. Students who engage in mathematical communication (via writing, speaking, reading, and listening) have the twofold benefit of communicating to learn mathematics and learning to communicate mathematically (NCTM, 2000; Pugalee, 2004). As students mature, their mathematics communication should also progress (NCTM, 2000). Students at increasing developmental levels acquire increasing mathematical abilities and knowledge; and in a similar manner, teachers should build on students' mathematics writing abilities as they progress through school (NCTM, 2000).

Further, in 2010, the National Governors' Association Center for Best Practices and Council of Chief State School Officers authored the Common Core State Standards in Mathematics (CCSS-M). The CCSS-M include eight Standards for Mathematical Practices (e.g., students reason mathematically and attend to precision) and they specify grade level criteria for mathematically proficient students. Common Core assessments tend to emphasize more writing across subject areas (Martin, 2015), and Common Core high-stakes tests require students to use more writing (Herbert & Powell, 2016). The authors of the CCSS-M support the NCTM's initiatives by maintaining students reason mathematically, construct viable arguments, and attend to precision (Martin, 2015). The CCSS-M standards include communicating precisely, using clear definitions, stating the meaning of symbols, and including units (Martin, 2015).

The Use of Writing about Solving Mathematics Problems in Mathematics

How Writing in Mathematics Helps Students

Many mathematics students think being successful in mathematics is just about getting the right answer or memorizing unconnected facts, but writing in mathematics helps students realize mathematics is more multi-dimensional (Braun et al., 2014; Van Dyke et al., 2014). Writing in mathematics helps students realize solving a mathematics problem is about the process (Knox, 2017; Santos & Semana, 2014), and it is not always just about getting the right answer or memorizing disjointed facts (Braun et al., 2014).

Students can also use writing in mathematics to make mathematical connections to their everyday lives (Baxter et al., 2005). This is because when students write and speak in mathematics, it builds connections to other areas in students' lives where they develop experiences and stories to make a personal narrative and develop their schema (Meier & Rishel,

1998). From this, students can ask questions, fill in gaps, and contextually shape their ideas and experiences (Meier & Rishel, 1998).

Over the course of one year, Baxter et al. (2005) studied how a seventh grade teacher used writing to support communication with low-achieving mathematics students. The authors analyzed student journals and identified numerous occasions where students used journal writing to explain their mathematical reasoning. From their study, Baxter et al. (2005) concluded writing enables students to have more ways to communicate in mathematics class and it gives students another way to show strategic competence and multiple representations (Baxter et al., 2005). It also helps students and teachers communicate in a more private venue (Baxter et al., 2005) where the student does not need to worry about being judged by classmates.

Writing in mathematics can help propel students deeper into the processes of exploring mathematics (O'Kelly, 2013), and it has been deemed as a way for students to reflect on their own mathematical processes (Pugalee, 2004). Students develop a sense of committed ownership and profound understandings (Braun, 2014). Students may begin to see mathematics as an area where they can develop opinions and they may see mathematics as a constantly developing subject (Braun, 2014) where they can connect new ideas to existing knowledge (Bangert-Drowns et al., 2011). For example, students may change how they feel about what they have learned, realize they understand more or less than what they thought, discern others' work, build on what they already know, and/or become more committed to mathematics (Braun, 2014). On page 37, Baxter (2008) notes mathematics writing "provides opportunities for the students and the teacher to pause and literally gather their thoughts."

Writing in mathematics helps students organize their knowledge, make sense of the question, and think more deeply (Braun, 2014; Cross, 2009). It provides a way for students to

communicate with others and themselves (Pugalee, 2004), and it encourages the writer to be explicit about what is happening internally (Peterson, 2007). Writing helps students consolidate their thinking, connect their ideas, and synthesize information to create a clean conceptual picture (Cross, 2009). For example, Cross (2009) found that students who write can better express their mathematical understandings. Eventually, writing in mathematics class may directly affect students' mathematical habits (Braun, 2014) by helping them see the "how and why of mathematics" (O'Connell et al., 2005, p. 198).

Yet Bangert-Drowns et al.'s (2004) meta-analysis of 48 school-based writing-to-learn programs concludes that writing has a minimal effect on traditional measures of academic achievement. Bangert-Drowns et al. (2004) acknowledge that the diverse characteristics of their study make it hard to make broad conclusions about the relationship between writing and learning. One limitation of this study was the authors only analyzed the effectiveness of one writing program. Another limitation of this study was that the authors just looked at traditional measures of academic achievement. They did not consider things that are not typically measured like metacognitive behaviors (Pugalee, 2001), problem solving strategies (Pugalee, 2004), and the long-term effects of writing in the classroom and how it might affect college and career readiness (Madison, 2012).

Cross (2009) studied the effects of writing alone, argumentation alone, and the combined effects of writing and argumentation in 211 ninth grade students and five teachers. Post-hoc analyses indicated that students who practiced either writing alone or writing and argumentation had significantly more mathematical gains than students in the control group (which did not participate in either arguing or writing; Cross, 2009). Students who participated in both writing and argumentation had significantly more gains than students in the argumentation alone and

control groups (Cross, 2009). Since learning outcomes for the writing alone group and argumentation and writing group were not significantly different, Cross (2009) concludes that "writing is a powerful strategy to promote learning" (p. 925).

Writing in mathematics may help students learn vocabulary (Bruun et al., 2015), and proper use of vocabulary is necessary for mathematics proficiency (NCTM, 2000). Writing about vocabulary allows students to think about the meanings of new words, put definitions in their own words (Bruun et al., 2015), and try using new vocabulary (NCTM, 2000). Stonewater (2002) found that higher scoring mathematics writers used more vocabulary. Students should use the language of mathematics to explicitly share their thinking (NCTM, 2000).

Students who succeed in mathematics have increased options for shaping their future (Dündar, 2016). Mathematics teachers would like their students to be mathematically proficient (Baxter et al., 2005). One measure of mathematical proficiency is the ability of individuals to become more expert in the practices of the mathematical community (Cross, 2009). Language can play an essential role in enabling an individual's ability to effectively engage in a community (Cross, 2009; McCormick, 2010). Communicating mathematical knowledge is a mathematics skill (Dündar, 2016; NCTM, 1998; NCTM, 2000).

How Writing in Mathematics Helps Teachers

When students communicate their thinking in mathematics, it clarifies their ideas and gives the teacher valuable information (Kenney et al., 2017; NCTM, 1989). Teachers can use mathematics communication to identify misconceptions (NCTM, 2000). According to Kenney et al. (2017), one pre-service teacher reported "the teacher can specifically see why that student got that problem wrong, and where they need help" (p. 40). Knox (2017) states teachers can look at students' writing to evaluate students' thinking process. For example, teachers can ask their

students to answer the prompt “explain how you know 1/4 is bigger than 1/5” (Kenney et al., 2017, p. 37). Writing in mathematics helps teachers recognize if their students actually know how to solve a problem (O’Connell et al., 2005). Additionally, it allows for genuine dialogue between students and the teacher about mathematical concepts (Braun, 2014; Dündar, 2016). Teachers can also use writing to justify planning decisions and inform their instruction (Baxter et al., 2005).

Writing in math can be used as a means of diagnostic, formative, or summative assessment. It can be used for diagnostic assessment when students provide personal writing about their mathematical experiences (Braun, 2014). For example, teachers can have students write an autobiographical excerpt about their mathematical experiences (Braun, 2014). As formative assessment, teachers can use it to adjust their instruction and give students feedback within the context of their own work (Santos & Semana, 2014). For example, students’ expository writing can be used to promote the development of mathematical reasoning and/or prompt mathematical concepts or procedures (Santos & Semana, 2014). It can be used for summative assessment when students are asked to summarize what they have learned about a concept (Knox, 2017).

Pugalee (2004) states despite encouragement from research and professionals, writing in mathematics classrooms has not gained wide acceptance. Porter and Masingila (2000) state “while writing has been praised as a useful tool for students’ learning, instructors who use writing in their classrooms also know that writing activities can mean longer hours spent reading students’ work and grading or responding in writing to it” (p. 174). Kenney et al.’s (2017) research article examines pre-service teachers’ pre- and post-conceptions of using writing to teach mathematics before and after they wrote in mathematics. Kenney et al. (2014) report

preservice teachers expressed a reluctance to use writing in the mathematics classroom because they did not see how it how it could be useful. But after some training, the pre-service teachers began to see it as a beneficial learning tool. Porter and Masingila (2000) call for further research to investigate the assumptions behind this topic.

Gillespie et al. (2014) analyzed the survey responses of 211 high school language arts, social studies, science, and mathematics teachers. On average, high school teachers in these subject areas spent 7.7 percent of their time writing, and most of this time students were filling in the blank. On page 1068, Gillespie et al. (2014) report most math teachers use "writing to solve a problem." Further, mathematics teachers were less likely (as compared to teachers in other disciplines) to discuss writing strategies with their students (Gillespie et al., 2014). The authors report the Common Core expects students to use more writing across all subject areas, but most high school teachers feel inadequately prepared to use writing in their classrooms (Gillespie et al., 2014). One limitation of Gillespie et al.'s (2014) study was that most of the respondents had master's degrees and averaged over 15 years of teaching experience, and teachers might have interpreted the survey questions in different ways.

How Writing in Mathematics Helps Facilitate Problem Solving

Writing activities can help students make sense of the question (Cross, 2009). According to Polya (1945), students first have to understand the question before they can solve it. This can happen through restating the problem and developing a plan (Polya, 1945). Writing helps students see larger patterns, evaluate their own ideas, remember what they have read, and sort out disagreements (Booth et al., 2008). In general, when people write they place more demands on themselves (Booth et al., 2008). This may be motivating because students may see their

writing as impacting someone else (Peterson, 2007). Writing in mathematics can enhance and extend classroom discussions (Baxter, 2008).

Craig (2016) investigated how writing explanatory paragraphs in mathematics class influenced problem solving behavior in 39 university calculus students. Polya (1945) defines problem solving as mathematical process where students understand the problem, develop a plan, implement the plan, and check their work. Craig (2016) mostly explored the first part of Polya's (1945) problem solving process. The author concluded that writing about problem solving in mathematics can cause a cogitative disequilibrium where students realize their reasoning is faulty or incomplete; and this in-turn, leads to a reflective process (Craig, 2016). Eventually, the writing process can propel the student to correct the learning deficiency (Craig, 2016).

Pugalee (2004) compared the problem solving process in ninth graders' written and verbal communication. If students got the wrong answers, Pugalee (2004) analyzed the type of error(s) made. The results of the study emphasized students who wrote descriptions of their problem solving processes produced significantly more right answers than students who processed verbally. This study highlights the relationship between problem solving and writing (Pugalee, 2004).

How Writing in Mathematics Helps Cross-Curricular Connections

Writing in mathematics not only applies to mathematics class, but it encourages quantitative reasoning across the curriculum. Students in health, sociology, economics, politics, sports, and education need to integrate mathematics into their everyday contexts (Kennedy et al., 2014; Madison, 2012). Writing in mathematics can strengthen arguments in all disciplines by adding "evidence, framing, focus, and precision" (Madison, 2012, p.3). This is because

quantitative reasoning provides evidence for more successful arguments in historical events, stories, politics, economics, journalism, and other areas (Madison, 2012).

Writing across the curriculum can help students learn about subject area concepts while helping them become better writers (Peterson, 2007). Content area writing provides authentic contexts for student writing (Peterson, 2007). The Common Core State Standards in English Language Arts (CCSS-ELA) emphasize that students provide reasoning and evidence to support their claims, produce clear and coherent writing to the specific task, gather relevant information, and routinely reflect on a range of tasks (Common Core Standards Initiative, 2010, p. 41). The CCSS-ELA also promote college and career readiness (Common Core Standards Initiative, 2010, p. 41). Writing in mathematics can be designed to address both CCSS-ELA and CCSS-M standards.

Hebert and Powell (2016) measured writing conventions in 155 mostly Caucasian English speaking fourth graders without disabilities. They analyzed students' introductions, conclusions, transitions, paragraphs, and the use of vocabulary to see how student use writing features to convey mathematical ideas (Herbert & Powell, 2016). The authors found that the fourth graders tended to use math symbols rather than words in their writing (i.e., "+" instead of "plus" and "=" instead of "equals") and students tend to reverse the subtrahend and minuend (Herbert & Powell, 2016). Additionally, Herbert and Powell (2016) report finding a lot of variability in how students use writing conventions in mathematics. The authors recommend explicit instruction for students to write in mathematics and advocate for future research regarding how students use writing conventions to express mathematical ideas (Herbert & Powel, 2016).

Applications of Writing in Mathematics at the University Level and in the Workplace

For almost 30 years, more writing at all levels of mathematics has been recommended in the United States by both the NCTM and Mathematical Association of America (Van Dyke et al., 2015). Yet Beaver and Beaver (2011) and Van Dyke et al. (2015) found that few students arrive at the university level from secondary school having been required and prepared to write mathematically; and by the time many students reach the college level, they feel it is inappropriate for them to write in mathematics class (Van Dyke et al., 2015).

Beyond the educational setting, knowing how to write in mathematics can be helpful in the workplace and to help improve public discourse (Madison, 2012). Many employers rank writing ability as one of the most important qualifications in the workplace, yet they are dissatisfied with many of their employees' writing skills (Quible et al., 2007). In 2004, poor writing abilities cost employers over three billion dollars annually in employee training (Quible et al., 2007). An example of improved public discourse with mathematics writing is a written clarification of a quantitative graphic (Madison, 2012). Additionally, if people wrote more with numbers, they might feel more empowered to question misuse of data and statistics (Madison, 2012).

Writing in Mathematics and Middle School Students' Comprehension

All students should be able to engage in meaningful mathematics communication (Baxter et al., 2005; NCTM 2000). Baxter et al. (2005) studied journal writing in seventh grade math classes and found writing in mathematics can be used to support lower achieving middle school students because it helps students who do not typically participate in class discussions take on a more active role in their learning (Baxter, 2008; Baxter et al., 2005). Lower achieving students

far too often spend much of their time with low-level practice; but writing in mathematics can help these students' develop conceptual understandings and problem-solving skills (Baxter et al., 2005). In the long term, writing in mathematics may even help low-level students who would otherwise avoid the subject develop longer lasting mathematical connections (Baxter et al., 2005).

Baxter (2008) studied mathematics writing with lower achieving 7th graders. Writing can give students more time to think, allow for multiple representations, and provide opportunities for students (especially quieter students) to communicate with the teacher. When students are stuck on a problem, they can write out their thought process and may be able to see their error and solve the problem (Baxter, 2008). Additionally, when students recognize their confusion, it is a step toward understanding. Baxter (2008) recommends teaching writing in mathematics thoughtfully and gradually.

Writing in mathematics can also be used to support gifted and talented students by pushing them further (Knox, 2017). Knox (2017) states teachers can analyze students' responses to evaluate each stage of the problem solving process. Diverse students benefit from different levels of teacher intervention (Polya, 1945). Teachers can model the writing process and provide their students with specific meaningful feedback so their gifted and talented students can see writing as a cognitive tool (Knox, 2017). Specific feedback can be used for differentiating instruction (McCormick, 2010.)

Bangert-Drowns et al.'s (2004) meta-analysis of 48 school-based writing-to-learn mathematics programs indicates that some writing tasks may be less effective for sixth through eighth graders. The authors speculate this could be because at this developmental stage, the

process of writing in different subject areas may seem very different (Bangert-Drowns et al., 2004).

Dündar (2016) researched the effect of writing in mathematics and increased success levels in secondary schools by analyzing quantitative data from 176 mathematics notebooks. Dündar (2016) found that more successful fifth, sixth, seventh, and eighth graders had better note taking skills. Kobayashi's (2005) meta-analysis adds that sixth through eighth graders had increased benefit from taking notes. Thus, Dündar (2016) concludes that meaningful and contextual writing directly affects student success.

Along the same lines, Santos and Semana (2015) studied expository writing in a group of eighth graders. The study analyzed interpretation, representation, and justification in students' formative assessments. The authors found that, with practice, eighth graders gradually increased their justifications and representations and decreased using vague representations (Santos & Semana, 2015). This study suggests that the combination of expository writing and feedback promote positive development in students' mathematical reasoning (Santos & Semana, 2015).

Textbooks also play a role in shaping how people write in math (Burton & Morgan, 2000). As students read, they see how author(s) organize their ideas and achieve the purpose of communicating their ideas (Peterson, 2007). Students can see how authors inform, persuade, explain, and entertain through writing (Peterson, 2007).

Van Dyke et al. (2015) observed that many university students are reluctant and some exhibit negative attitudes toward writing in mathematics class. They predict that if middle and high school students practiced more writing in mathematics classes, then these experiences might possibly have a boosting impact on their attitude toward mathematics writing at the university level (Van Dyke et al., 2015). This may be especially true if positive conceptual mathematics

writing experiences are fostered on a regular basis in earlier grades (Van Dyke et al, 2015). In the long run, both teachers and students have much to gain from this process (Van Dyke et al., 2015).

The Use of Writing in Mathematics and Students' Metacognition

Metacognition is defined as a flexible cogitative strategy where students regulate and have a knowledge of their own cognition (Paris & Paris, 2001; Schraw 1998). Metacognitative awareness applies to declarative, procedural, and conditional knowledge and it involves planning, monitoring, and evaluating (Schraw, 1998). Developing mathematical conceptual knowledge is likely bidirectionally linked to metacognition (Pugalee, 2001). This is because metacognition involves employing appropriate information and strategies during the problem solving process (Pugalee, 2001).

Metacognition is linked with increased success in student learning situations (Pugalee, 2001). Research indicates metacognitive processes can help students achieve mathematically and promote higher-level thinking (Cross, 2009), and it is considered one of the most important factors in student learning (Knox, 2017). Developing metacognitive skills and discourse in the classroom have been tied with students having a deeper understanding and increased mathematical achievement (Cross, 2009). The NCMT (2000) notes teachers can use written communication as a way to have students reflect on what they learned in a lesson or on what remains unclear.

Knox (2017), Pugalee (2004), and Cross (2009) agree that writing can be an effective metacognitive tool, and writing is one way to increase students' metacognition (Knox, 2017).

This is because writing involves individuals using their cogitative abilities to generate information (Cross, 2009). Knox (2017) states reflective mathematics writing can help students decipher strategies for solutions and recognize strengths and areas of improvement. However, Bangert and Drowns et al. (2004) caution that not all mathematics writing will improve metacognitive awareness.

Writing Strategies in the Mathematics Classroom

Learning mathematics usually consists of students competing problems to reinforce the days' lessons (Braun, 2014). Following this pattern may result in a lack of critical thinking (Braun, 2014). For example, students may not understand why pi is approximately 3.14 (Braun, 2014); understand why the Pythagorean Theorem is true (Braun, 2014); or understand when to use which measure of central tendency. Braun (2014) advocates for mathematics students to do more than just show what they understand with tests, quizzes, and homework. Teachers can use writing in mathematics to have students include details, explain thoroughly, and think more deeply and clearly about concepts (Braun, 2014).

Writing in mathematics class may be expository, critical, personal, or creative (Braun, 2014). Possible mathematical writing assignments include journals, explaining how to solve math problems, explaining mathematical ideas, and writing about learning processes (Burns, 2004; Knox, 2017). Other activities include students writing definitions in their own words, writing about problem solving stages, writing summative reflections over the topics studied, and writing a mathematical biography (Knox, 2017). Peterson (2007) suggests students write question-answer poems about problems and their solutions. Further ideas include using student

writing in subsequent lessons, having students discuss their ideas before writing, posting vocabulary, and having students take on writing assignments in groups (Burns, 2004).

Quality writing in mathematics may improve when students are allowed to share their work because group writing projects help students communicate mathematically (Wilcox and Monroe, 2011). The teaching strategy "think-write -share" may heighten students' engagement in writing while synchronously holding them accountable for their learning (Wilcox and Monroe, 2011). Or students can respond to each other's work within a "writing circle" (p. 3), where they give positive feedback and clear suggestions (McCormick, 2010). Wilcox and Monroe (2011) recommend teachers have students write class books. Along with providing opportunities to revise and edit, class books provide a sense of audience (Wilcox & Monroe, 2011). However, Wilcox and Monroe (2011) warn that writing with revision will likely take more class time.

Teachers may worry that with more class time spent on writing, there will be less time for mathematics topics (McCormick, 2010). This may be mitigated through teachers assigning increasingly more complex topics for student writing projects (McCormick, 2010). Such topics could extend deeper into mathematics than class time typically allows (McCormick, 2010). Thus, the time expenditure of the two would balance each other out. Peterson (2007) adds that teachers may be able to use both Language Arts and subject area class time to work on content area writing projects.

Writing in mathematics can take on many types. Depending on the type of writing, the subject matter, social functions, and the role of author and reader can vary (Burton & Morgan, 2000). "Writing mathematically is similar to writing in any genera because it needs practice" (NCTM, 2000, p. 62). Yet little attention has been paid to how novices learn to write mathematically (Burton & Morgan, 2000). McCormick (2010) recommends teachers highlight

and evaluate exemplar writing pieces. While O'Connell et al. (2005) suggest teachers help students organize their thoughts before writing, demonstrate how to put ideas into writing, and provide them with the same supports as when students write in other content areas.

It is of utmost importance that teachers develop social norms for writing in their classroom (Cross, 2009). When teachers use writing in their mathematics classroom, Braun (2014) recommends teachers start slowly and set clear expectations. Bangert-Drowns et al. (2004) add that expectations associated with writing affect the usefulness of the practice. Cross (2009) states that teachers should continuously scaffold and regularly model mathematical discourse. Students in lower grades will need more instructional support from their teachers, and by the time they get to grades 3-5 they should gradually learn to become more independent (NCTM, 2000). With increasing grade levels, mathematical arguments should become more rigorous (NCTM, 2000). Beginning in middle school, students should be able to write about their mathematical ideas in a formal manner with mathematical terminology (NCTM, 2000). High school students should be able to critique themselves and others (NCTM, 2000). They should be able to generate questions, explanations, and correct mathematical arguments (NCTM, 2000).

Middle school teachers should create a classroom environment were students feel a sense of community (NCTM, 2000). Students should feel like they can communicate honestly without the fear of derision (NCTM, 2000). They should feel like they can seek clarification until they fully understand the concept (NCTM, 2000). In such a communication-rich community, students should feel that it is okay to struggle and make mistakes (NCTM, 2000).

Teachers should guide their students to become better writers in mathematics (NCTM, 2000). Dündar (2016) states that part of the teacher's duty is to encourage students to write, encourage students to have positive attitudes towards writing, help students who have difficulties

writing sustain their writing, and help students develop cognitive skills through writing. Teachers should ask their students to include details and explain thoroughly (Burns, 2004). Teachers should pay special attention to mathematical argument, mathematical representations, and standards of explanation and proof (NCTM, 2000). Additionally, teachers should help their students sequence ideas and add details (NCTM, 2000).

Since many students struggle with clarifying their mathematical thinking in writing, Stonewater (2000) recommends teachers provide a framework for helping students learn to write in math. This may take the form of a checklist (Stonewater, 2000). Teachers can help struggling writers in mathematics by demonstrating good and poor examples, incorporating peer feedback, providing students with specific feedback, and letting students practice.

Chapter Two Summary

Writing in mathematics can help both students and teachers in many ways. It helps students see the "how and why of mathematics" (O'Connell et al., 2005, p. 198). It helps teachers evaluate students' thinking processes. It can help facilitate problem solving, be used to make cross-curricular connections, and it can prepare students for the workplace and/or the university. Additionally, writing in mathematics can be used to enhance metacognition. Teachers should provide a communication-rich classroom environment, provide scaffolding and explicit mathematics writing instruction, and give their students opportunities to practice.

Chapter Three: Response to Research Questions

Reflection on the Literature

The literature tells us a considerable amount about the use of writing when solving mathematics problems in mathematics class. Writing in mathematics has the potential to give the teacher valuable information (Kenney et al., 2017), express students' mathematical understandings (Cross, 2009), help students see mathematics as a process (Knox, 2017; Santos & Semana, 2014), and connect students' mathematical ideas (Cross, 2009). Additionally, it can be used for diagnostic, formative, and summative assessment. Stonewater (2000) reports that, in general, when writing is used, students tend to have greater mathematical ability.

Writing in mathematics can affect both lower- (Baxter et al., 2005) and higher-achieving (Knox, 2017) middle school students' mathematics comprehension. Writing in mathematics can help lower-achieving students' develop conceptual understandings and problem solving skills (Baxter et al., 2005). Knox (2017) states writing in mathematics can help higher-achieving students by pushing them further and as a means of differentiated instruction. Writing in mathematics may help middle school students who do not typically participate in class take on a more active role in their learning (Baxter, 2008; Baxter et al., 2005). Kobayashi (2005) adds that sixth through eighth graders have increased benefits from taking notes, and Dündar (2016) concludes that meaningful and contextual writing directly affects middle school student success. For example, Santos and Semana (2015) found that with practice, the combination of expository writing and feedback promotes growth in middle schoolers' mathematical reasoning.

The literature tells us much about how the use of writing in mathematics affects students' metacognition, and metacognition has been deemed one of the most important factors in student learning (Knox, 2017). Developing metacognitive skills and discourse in the classroom have been linked with students having a deeper mathematical understanding and increased mathematical achievement (Cross, 2009). Several researchers agree that writing can be an effective metacognitive tool, and writing is one way to increase students' metacognition (Knox, 2017; Pugalee 2004). Knox (2017) states reflective mathematics writing can help students decipher mathematics strategies and recognize strengths and areas for improvement.

Classroom Implementation

In 1989 and 2000, the NCTM highlighted the importance of teachers creating a classroom environment where students feel they can communicate their mathematical thoughts both verbally and in writing without judgement (NCTM, 1989, NCTM 2000). Students should feel they can test their understandings on the basis of shared knowledge (NCTM, 2000). This helps students' communication skills (NCTM, 2000; Pugalee, 2004). Developing an emotionally safe classroom environment has always been an important part of my teaching practice, and it is one of the reasons why I became a teacher. Both parents and my principal have commented on how students seem to feel comfortable communicating and participating in my classroom. To foster a positive communication-rich classroom, I use several strategies:

- Students go to the board (if they wish) and complete daily bell work problems. This is followed by positive feedback where other students tell the student who went to the board what they like about their solution.

- Students go to the document camera (were the teacher usually sits) to solve problems in front of the whole class.
- Students play a variety of mathematics games.
- Students give silly claps (like a "marshmallow clap," "alligator clap," or "snow clap") to each other when students solve a mathematics problem in front of the class.
- The poster in the front of my classroom says "Everyone is Welcome Here, Everyone Belongs."
- Students participate in "number talks," where students describe their problem solving strategies to the rest of the class. Other students say if they solved the problem the same way or a different way.
- I go over the most missed math homework questions with the class, and talk about common mistakes.

The NCTM (1989 & 2000) publications imply that writing in mathematics classrooms should coincide with communication-rich classroom environments. In communication-rich learning communities, students should feel like it is okay to struggle and make mistakes (NCTM, 2000). The last three years, I have made an effort for students to see making mistakes as a normal part of the learning process. One of my students even commented to the whole class that they liked how a fellow student made a mistake at the board and fixed it. To foster a classroom where students feel they can make mistakes (and fix them), I use several strategies:

- I emphasize that we do math in pencil because it is normal to make mistakes.
- I make mistakes when I solve problems in front of the class, and after I discuss how "my brain grew" from making a mistake.

- I provide the class with several problems with wrong answers and their job is to fix the problems and identify each mistake.
- I regularly discus how it is okay to make mistakes because making mistakes helps us learn.

Before embarking on this literature review, I viewed students writing in the mathematics classroom and the classroom environment as two discrete areas. I had not considered how the classroom environment would impact students' writing or how writing would impact the classroom environment. However, the NCTM (1989 & 2000) and Cross (2009), stress the importance of classroom environments and student communications in mathematics class. For example on page 909, in regard to community practices and classroom communication, Cross (2009) highlights the importance of teachers promoting "valid but diverse ways of thinking." Further, on page 62, the NCTM (2000) states, "middle school teachers should create a sense of community in their classrooms where students can communicate honestly, openly, and without fear of ridicule. Students should have a communication-rich classroom where they can seek clarification until they feel like they understand."

The NCTM (2000) recommends teachers guide their students to become better writers, and they should pay special attention to mathematical argument and mathematical representations. Additionally, teachers should help their students sequence ideas and add details (NCTM, 2000). I provide scaffolding and modeling for my students as they learn how to write in mathematics. For example, I provide a fill-in-the-blank format for students at the beginning of the year and they transition to a list of steps to help them write. Throughout the year, I provide feedback to help them develop mathematical arguments, sequence ideas, and add details. This year, I have been working with a professional learning community at my school to improve

writing across grade levels and subject areas. When my students complete a writing assignment, they are encouraged to use skills from Language Arts to help their writing process.

Many people view the writing process as a work in progress where editing, revising, and rewriting are normal and common practices. Usually, written work is never perfect the first time. Likewise, it is okay not to be perfect the first time when solving a mathematics problem. Yet many students think mathematics is all about getting the right answer (Van Dyke et al, 2014), and many want the answer to be perfect the first time they solve a problem. Perhaps if students write more in mathematics, they will begin to see mathematics as an area where they can make revisions, edit and revise. Students might possibly realize they do not always have to be right the first time when they solve a math problem. Like the writing process, mathematics problems could require editing, revising, and resolving a problem. Eventually, students could view this as a normal part of their learning process.

This literature review inspired me to ask writing teachers at my school how they reinforce the idea of editing and revising with their students:

- The middle school Language Arts teacher said he uses peer review/writer's workshops. In this process, the author reads his or her work out loud. This helps the students self-edit and receive critique from others. The Language Arts teacher believes this is an essential step in determining if the writing truly communicates what it is intended to communicate. He also comments on their drafts by offering a couple of "glows" (things they have done well) and a couple of "grows" (things that need improvement).
- The fifth grade Language Arts teacher has the students revise in one color and edit in a different color. Her reasoning behind this to help students focus on little bits at a time. She also has them read the paper out loud to another classmate or to her.

- The fourth grade teacher provides writing models for her students. She also goes through the editing and revising process with different colors of writing utensils. Considering only one sentence at a time, fourth graders review their writing with a partner.
- The second grade teacher said she provides different supports depending on the time of year. At the beginning the year, she and the students edit a sample writing piece. As the year progresses, students get their own rubrics and check off writing conventions like capital letters and punctuation. Weekly, students find mistakes as a class in a paragraph and relate these mistakes to their own writing. One of her goals is to help second graders feel confident in their writing.

This literature review also inspired me to have my students invest more time editing and revising; and it inspired me to have my students do more mathematics writing activities in small groups. For example, I had students give each other feedback within a "writing circle" (McCormick, 2011) or through peer editing. This gives students opportunities to reflect on their own strengths and weaknesses and helps them discern different ways people think about the same mathematical problem (McCormick, 2011). When my students did this activity, they told me that the Language Arts teacher at my school does something similar. Both the Language Art teacher and I found that students did not seem to be motivated to peer edit each other's' writing. Conner and Moulton (2000) also noticed a lack of motivation from 8th grade students to peer edit. One way Conner and Moulton (2000) motivated their students to write well was to have them write a class book.

Writing a class book provides a sense of audience, and it allows students to communicate mathematically while providing opportunities to revise and edit (McCormick, 2011). I was also

inspired by Wilcox and Monroe (2011), and I plan to have my students write a class book about mathematical concepts they have learned throughout the school year.

My classroom experience with fifth graders confirms several of Herbert and Powell's (2016) findings with fourth graders. Herbert and Powell (2016) found that many students reverse the subtrahend and minuend in their mathematics writing. Additionally, I have found that many students switch the dividend and divisor in their writing. When my students do this, I provide feedback on how they can place them in the correct order. Hebert and Powell (2016) also found that fourth graders tended to use mathematics symbols rather than words in their writing. Indeed, many of my students use mathematics symbols in their writing. I regularly give them feedback to use more words, but Herbert and Powell's (2016) endorsement of explicit instructions to have students write in mathematics inspired me to directly instruct all of my mathematics classes to use fewer mathematics symbols in their writing and to give them more generalized writing instruction.

Stonewater (2000) advises mathematics teachers to provide a framework adapted to the students' learning situation to help them write. In my experience, giving students steps to use when writing a math paragraph helps facilitate their writing process. They can think more deeply about the mathematics behind what they are saying rather than worry about how they say it. For example, I had one student write a paragraph without his steps because he lost them; and I gave him the steps for his next paragraph. The paragraph he wrote with steps was much improved and received a much higher grade. The student demonstrated he understood the concept; it made sense, and it was connected to what he had learned in mathematics class.

My sixth grade mathematics class wrote an expository paragraph explaining how to convert 7.0×10^2 from scientific notation to standard form. When I taught this concept to the

class, I emphasized how you can just move the decimal over two places to the right (because the exponent is two) to make the number 700. However, in their paragraphs, several of my students wrote 10^2 equals 100 (and/or 10 times 10) and then you should multiply 100 by 7 to get 700. This showed me the class understood the concept on a deeper level than I anticipated and on a deeper level than I emphasized in class. Additionally, I was delighted that each student showed a thorough understanding of the concept through their writing, and the whole class could advance beyond this concept. Later when I taught scientific notation with small numbers, I linked the lesson to their paragraphs and discussed how 10^{-2} equals $\frac{1}{10*10}$ or one hundredth.

Baxter (2008) and Baxter et al. (2005) state writing in mathematics class can help students take on a more active role in their learning. Before one of my math classes embarked on a "silent conversation," I had a student get pulled out of the classroom. At the beginning of the year, other teachers told me it was really hard to get him to write because he did not like writing. As this student was leaving the room, he commented on how he was disappointed because he really wanted to participate in the silent conversation and did not want to miss this part of class. He asked me if the class would do it again when he would be there, and I assured him we would. Another student told me writing mathematics paragraphs was her favorite part of mathematics class, and she felt successful doing it.

Gillespie (2014) recommends teachers discuss with their students why a writing activity is effective. This prompted me to spontaneously ask my Pre-Algebra class students why I have them write. Before asking them, I did not say anything about why I think writing is important or what I found in the literature. Several students contributed to our class discussion:

- When you write it, it helps you remember it better.
- It allows you to communicate it.

- It allows you to understand better.
- It shows the teacher you understand it, and if not, it lets the teacher know what to teach again.
- It makes it harder; and is a different way to show what you know.

As they were sharing their ideas, I was astonished by how their thoughts coincided with how the literature says writing in mathematics is beneficial. However, my students had not read peer reviewed journal articles. They were speaking purely from their experiences of writing in my mathematics classes.

Bangert-Drowns et al.'s (2004) meta-analysis found the use of metacognitive prompts can enhance the effect of writing in mathematics; and the NCTM (2000) encourages the use of worthwhile writing tasks. The writing prompt should be well thought out by the teacher, and it should help the student reflect on their current knowledge, misperceptions, and learning processes (Bangert-Drowns et al., 2004). I asked my students to write a paragraph comparing and contrasting 2^{-3} and $2.0 \text{ X } 10^{-3}$. I chose this topic because in my years of teaching, I observed many students having trouble differentiating between these two concepts; and I wanted to design a writing prompt that would show me how my students were thinking about their current knowledge. Students' written responses accentuated strategies they used to differentiate between the two expressions. For example, several of my students wrote the answer to 2^{-3} is usually in fraction form and the answer to $2.0 \text{ X } 10^{-3}$ is usually in decimal form. Additionally, one of my students wrote the answer to 2^{-3} is eight instead of one eighth.

Recommendations and Possible Areas of Future Study

Bangert-Drowns et al.'s (2004) meta-analysis of 48 school-based writing programs found that writing in mathematics had a minimal effect of measures on traditional measures of academic achievement. They also reported the writing prompt and implementation of the writing assignment make a difference in the effectiveness. One limitation of this study is the authors only measured traditional measures of academic achievement. The authors did not consider other measures of academic achievement like college and career readiness (Madison, 2012), metacognitive behaviors (Pugalee, 2001), problem solving strategies (Pugalee, 2004) or development of mathematical reasoning. On page 31, Bangert-Drowns et al. (2004) concede given the limitations of their study, "it is hard to arrive at precise conclusions about the relationship between writing and learning." Teachers may be able to mitigate problems reported by Bangert- Drowns et al. (2004) through scaffolding, creating a positive communication-rich classroom environment, and helping students see writing as a normal part of mathematics class.

When researchers consider the effectiveness of writing in the mathematics classroom, they should recognize that its usefulness depends on contextual factors (Bangert-Drowns et al., 2004; Cross, 2007). For example, the learning benefits of writing in mathematics may increase as students become more accustomed to the writing process (Bangert-Drowns et al., 2004; Santos & Semana, 2015) and when students are in communication-rich classrooms (NCTM, 1989; NCTM, 2000). This supports the NCTM's affirmation that communication in the mathematics classroom is affected by classroom environment (NCTM, 1989; NCTM 2000).

Little attention has been paid to how novices learn to write in various mathematical genera (Burton & Morgan, 2000). Like writing in other subject areas, writing in mathematics takes practice (NCTM, 2000). More research needs to be done on what kind of mathematics

writing benefits students the most (Van Dyke, 2015). Several authors (Bangert-Drowns et al., 2004; Santos & Semana, 2015; Van Dyke, 2015) conclude that some writing assignments are more beneficial than others. For example, Santos and Semana (2015) found that expository writing helped improve middle schoolers' justifications and interpretations. Perhaps different students benefit more from some types of writing than others. For example, some students may benefit more from writing a mathematical autobiography or from journaling, while other students may benefit more from writing a class book or from explaining how to solve a problem. Additionally, some writing assignments may be more beneficial for some age groups than others.

Summary of Findings and Opportunities for Further Implication

When considering writing in the mathematics classroom, researchers should consider how it impacts college and career readiness, development of mathematical reasoning, development of problem solving behaviors, and metacognitive behaviors. Its usefulness depends on contextual factors like scaffolding, regular practice, and a communication-rich environment. Writing in mathematics can positively affect both high- and low-achieving middle school students. In addition to what I am already doing in my middle school mathematics classroom, I would like to further implement mathematics writing by:

- Providing more opportunities for students to edit.
- Assigning different types of group writing activities, like having students write a class book.
- Reinforcing writing instruction that has already been provided by other teachers.
- Providing more explicit instruction on writing in mathematics.

- Developing more writing prompts that promote metacognition.
- Building on the scaffolding and assessment already in place in my classroom.

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Author's Biography

After growing up in Wyoming, Kristy M. Palmer got her Bachelor of Science degree in Biology from Furman University in Greenville, South Carolina. She finished her first Master of Science degree on selenium bioavailability in pikas from the University of Wyoming's Zoology and Physiology Department. After earning these two degrees, she was a Biology instructor at Laramie County Community College and wildlife biologist in Wyoming where she studied several species. After working as a biologist for several years, she decided to go back to the University of Wyoming for her teaching certification in secondary biology. This was followed by additional teaching certifications in middle school science, K-12 health, and middle school math. She joined the University of Wyoming's Science and Math Teaching Center for her middle school math certification; and this thesis is a part of that same program. While she was in graduate school the second time, she participated on the University of Wyoming's Nordic Ski team where she competed at Nationals. She is currently a middle school science and math teacher at Snowy Range Academy (Wyoming's first charter school) in Laramie, Wyoming.