Perturbation Kernels for Generalized Seismological Data Functionals (GSDF)

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SUMMARY

In seismic waveform analysis and inversion, data functionals can be used to quantify the misfit between observed and model-predicted (synthetic) seismograms. The generalized seismological data functionals (GSDF) of Gee & Jordan quantify waveform differences using frequency-dependent phase-delay times and amplitude-reduction times measured on time-localized arrivals and have been successfully applied to tomographic inversions at different geographic scales as well as to inversions for earthquake source parameters. The seismogram perturbation kernel is defined as the Fréchet kernel of the data functional with respect to the seismic waveform from which the data functional is derived. The data sensitivity kernel, which is the Fréchet kernel of the data functional with respect to structural model parameters, can be obtained by composing the seismogram perturbation kernel with the Born kernel through the chain rule. In this paper, we extend GSDF analysis to broad-band waveforms by removing constraints on two control parameters defined in Gee & Jordan and derive the seismogram perturbation kernels for the modified GSDF analysis. The modifications given in this paper are consistent with the original GSDF theory in Gee & Jordan around the centre frequency and improve the stability of GSDF analysis towards the edges of the passband. We also present numerical examples of perturbation kernels for the modified GSDF analysis and their data sensitivity kernels using a homogenous half-space structure model and a complex 3-D structure model.

Key words: Time series analysis; Inverse theory; Seismic tomography; Computational seismology.

1 INTRODUCTION

Advances in parallel computing technology and numerical methods have made large-scale, 3-D numerical simulations of seismic wavefields much more affordable and they open up the possibility of ‘full 3-D tomography’ (F3DT), in which the starting model as well as the derived model perturbation is 3-D in space and the Fréchet kernel is computed using the full physics of 3-D wave propagation. Two physically equivalent but computationally distinct approaches to F3DT (Chen et al. 2007a) have been developed, the scattering-integral (SI) method, which sets up the inverse problem by calculating and storing the Fréchet kernels for individual misfit measurements (Zhao et al. 2005) and the adjoint-wavefield method, which constructs the gradient of the objective function through correlating the forward wavefield from the source and the adjoint-wavefield from the receivers (Tarantola 1986; Tromp et al. 2005). The first successful application of F3DT using real data from natural earthquakes was conducted in Chen et al. (2007b) to improve the 3-D crustal structure in the Los Angeles Basin region using the SI method. Recently, Tape et al. (2009) has adapted the adjoint-wavefield method to image the crustal structure in Southern California using waveform data from local earthquakes and Fichtner et al. (2009) has adapted the adjoint-wavefield method to continental-scale tomography and inverted for upper-mantle structure in the Australasian region. In these successful F3DT applications, time- and frequency-dependent phase and amplitude anomalies were used to quantify the misfit between synthetic and observed seismograms.

A data functional is a map that assigns each member of a certain class of earth models a single observable number that can be extracted from the seismograms. Given an earth model, we can compute synthetic seismograms by solving the seismic wave equation and the discrepancies between the synthetic seismogram and the corresponding observed seismogram could be quantified using data functionals. Examples of data functionals include the differential travel-time measured by maximizing the cross-correlation between the observed and synthetic waveforms (Woodward & Masters 1991) and the time- and frequency-localized phase-delay and amplitude anomalies (Thomason 1982; Gee & Jordan 1992; Laske & Masters 1996; Ekström et al. 1997; Holschneider et al. 2005; Kristekova et al. 2006; Fichtner et al. 2008; Fichtner et al. 2009; Kristekova...
et al. 2009). The Fréchet kernel of a data functional with respect to structural parameters can be obtained by composing the Born kernel (Dahlen & Tromp 1998), which provides the Fréchet kernel of the waveform with respect to structural parameters and the seismogram perturbation kernel, which is the Fréchet kernel of the data functional with respect to the waveform (Chen et al. 2007a), through the chain rule (Milne 1980, p. 293).

The generalized seismological data functionals (GSDF) of Gee & Jordan (1992) quantify waveform differences using frequency-dependent phase-delay times and amplitude-reduction times measured on time-localized arrivals. 1- and 2-D data sensitivity kernels for GSDF measurements were derived in Gee & Jordan (1992), Zhao & Jordan (1998) and applied in several tomographic inversions (e.g. Gaherty et al. 1996; Katzman et al. 1998). 3-D data sensitivity kernels for GSDF phase-delay measurements were first presented in Zhao et al. (2000) for a 1-D starting model by expressing the Born kernel as a coupled-mode summation and later applied to investigate 3-D seismic structure of the mantle beneath western Pacific (Chen et al. 2002). In this paper, we extend GSDF analysis to broad-band waveforms by removing constraints on two control parameters defined in Gee & Jordan (1992): the pre-filtering parameter $\tilde{g}_{\omega}$ and the frequency-shift parameter $\tilde{\xi}_{\omega}$. We also derive the perturbation kernels for the modified GSDF analysis and give a noise model for the GSDF measurements based on the derived perturbation kernels. In our formulation and through numerical examples, we show that the modifications introduced in this paper are consistent with the original GSDF theory in Gee & Jordan (1992) around the centre of the frequency band and improve the stability of GSDF analysis towards the edges of the passband. The formulation given in this paper has been successfully applied in the F3DT for the Los Angeles Basin region in Chen et al. (2007b).

2 SEISMOGRAM PERTURBATION KERNEL

Seismogram perturbation kernels are defined as the Fréchet kernels of data functionals with respect to the seismic waveforms from which the data functionals are derived. We presume the seismogram observed on the $i$-th component of the $r$-th receiver from the $s$-th seismic source can be approximated by an instrument-filtered displacement $\tilde{u}_i^r(x, t)$. For each seismogram, we consider a finite set of data functionals $d_i^r$, indexed by $n$, that measure the misfit between $\tilde{u}_i^r(x, t)$ and the instrument-filtered displacement $u_i^r(x, t)$ synthesized from the starting earth model $m_0$:

$$d_i^r = D_n[u_i^r(x, t), \tilde{u}_i^r(x, t)].$$

(1)

The measurement process, denoted as $D_n$ in eq. (1), generally involves non-linear operations on both the observed and synthetic seismograms. We assume it is constructed to satisfy non-linear operations on both the observed and synthetic seismograms. We assume it is constructed to satisfy

$$\delta d_i^r = \int dt J_i^r(t) \delta u_i^r(x, t).$$

(2)

where $\delta u_i^r(x, t)$ is the displacement perturbation. The exact Fréchet derivative of the displacement with respect to structural parameters is provided by the first-order Born approximation (Dahlen & Tromp 1998; Zhao et al. 2000). The Fréchet derivative of the measurement operator $D_n$ with respect to displacement is a linear integration operator with an integration kernel given by $J_i^r(t)$ and maps a perturbation in the displacement to a perturbation in the data functional.

The integration kernel $J_i^r(t)$, which accounts for the effects of measurement operator on the target waveform as well as any instrument filtering, is what we call the seismogram perturbation kernel. Examples of seismogram perturbation kernels of some widely used data functionals are given in Chen et al. (2007a).

3 GENERALIZED SEISMOLOGICAL DATA FUNCTIONALS

The GSDF method (Gee & Jordan 1992) provides a unified framework for the analysis and inversion of broad-band waveform data. In the frequency domain, we can map the synthetic waveform $u_i^r(x, \omega)$ into the observed waveform $\tilde{u}_i^r(x, \omega)$ using two frequency-dependent, time-like quantities $\delta \tau_i^r(x, \omega)$ and $\delta \omega_i^r(x, \omega)$.

$$\tilde{u}_i^r(x, \omega) = u_i^r(x, \omega) \exp[i\omega(\delta \tau_i^r(x, \omega) + i\delta \omega_i^r(x, \omega))].$$

(3)

In GSDF analysis, we estimate $\delta \tau_i^r(x, \omega)$ by measuring frequency-dependent phase-delay time $\delta \tau_i^r(\omega)$ and amplitude-reduction time $\delta \omega_i^r(\omega)$ at a set of discrete frequencies of interest $\omega_i$.

The GSDF data processing consists of several steps (Fig. 1). We isolate the target wave group using an isolation filter $1(t)$, which is obtained by windowing the complete synthetic seismogram. We then cross-correlate the isolation filter with the complete synthetic seismogram and with the observed seismogram and window the resulting synthetic and data cross-correlagrams around the zero-lag. The windowed correlagrams are then narrowband filtered at a set of frequencies $\omega_i$. When certain conditions about windowing and narrowband filtering are enforced (Gee & Jordan 1992), the resulting narrowband-filtered windowed correlagrams can always be well matched by five-parameter Gaussian wavelets, which are cosine functions with frequencies at around $\omega_0$ and modulated by Gaussian envelopes. The differences in the phase and the amplitude between the synthetic and data Gaussian wavelets give us the phase-angle $\omega_0$ at each narrowband-filtering frequency $\omega_i$. A practical issue of the GSDF analysis is that the phase-delay measurements need to be corrected for possible cycle-skipping errors before they can be used in inversions. These cycle-skipping errors can usually be corrected by bootstrapping the phase from low frequencies to high frequencies (Ekström et al. 1997).

In traditional broad-band cross-correlation analysis, the traveltime shift $\Delta \tau$ of an isolated waveform is estimated using the location of the cross-correlagram peak (Luo & Shuster 1991; Woodward & Masters 1991; Dahlen et al. 2000; Zhao et al. 2000) and the amplitude anomaly can be determined from the maximum amplitudes of the cross-correlagrams (Dahlen & Baig 2002; Ritsema et al. 2002). For band-limited signals, these measurements provide good estimates around the dominant frequency but they do not characterize the differences in the shape of the waveforms. In the GSDF analysis, we can account for differences in waveform shapes by making measurements at several frequencies across the frequency bandwidth. In Fig. 2, we illustrate this point using an example. By correcting the phase and amplitude of the synthetic waveform using the GSDF measurements made at five frequencies evenly distributed over the frequency band, we were able to recover the observed waveform almost perfectly.

The GSDF measurements are well suited for tomographic inversions. In particular, their linearization depends on the Rytov approximation, which is valid for large phase-shifts as long as the phase perturbation per wavelength is small (Chernov 1960; Snieder & Lomax 1996). This is far less restrictive than the Born approximation, which requires small phase-shifts.
Following Gee & Jordan (1992), we first apply a Gaussian time window of the form

\[ W(t) = \exp\left[-\frac{\sigma^2}{2}(t-t_c)^2\right], \]

onto both correlagrams to reduce the contributions of interfering wave groups to the observations. For the symmetric autocorrelation in eq. (4), we set \( t_c = 0 \). In general the cross-correlation in eq. (5) is not symmetric and to minimize the signal distortion by windowing we usually centre the time window at the peak of the cross-correlagram. The windowed correlagrams are denoted as \( \tilde{W}C_{ff} \) and \( WC_{ff} \).

The next operation is the narrowband filtering which localizes the windowed correlagrams in the frequency domain. We consider Gaussian narrowband filter of the form:

\[ F_i(\omega) = \exp\left[-\frac{(\omega - \omega_i)^2}{2\sigma_i^2}\right] + \exp\left[-\frac{(\omega + \omega_i)^2}{2\sigma_i^2}\right], \]

where the index \( i \) specifies a filter \( F_i \) with half-bandwidth \( \sigma_i \) and centre frequency \( \omega_i \). After some algebraic manipulation (Chen 2005), we obtain the windowed and narrowband filtered correlagrams in time domain:

\[ F_iWC_{ff}(t) = \exp\left[-\frac{(\omega_i^1)^2}{2}\right] \times \sum_{n} \left[ A_n^1 \cos[\omega_n^1(t + \tau_p^1)] + A_n^2 \cos[\omega_n^2(t + \tau_p^2)]\right], \]

\[ F_iWC_{ff}(t) = \exp\left[-\frac{(\omega_i^1)^2}{2}\right] \times \sum_{n} \left[ A_n^1 \cos[\omega_n^1(t + \tau_p^1)] + A_n^2 \cos[\omega_n^2(t + \tau_p^2)]\right]. \]
Here,
\[ \sigma_i' = \sqrt{\frac{\sigma^2 - \sigma_i^2}{\sigma_i^2 + \sigma_c^2}}. \] (10)

\[ \omega_a' = \frac{\sigma^2 \omega_a + \sigma_c^2 \omega_i}{\sigma_i^2 + \sigma_c^2}. \] (11)

\[ \omega_a'' = \frac{\sigma^2 \omega_a - \sigma_c^2 \omega_i}{\sigma_i^2 + \sigma_c^2}. \] (12)

\[ \tilde{A}_n = \tilde{A}_n \frac{\sigma_i'}{\sigma_w} \exp \left[ -\frac{(\omega_a - \omega_i)^2}{2(\sigma_i^2 + \sigma_c^2)} \right]. \] (13)

\[ \tilde{A}_n' = \tilde{A}_n \frac{\sigma_i'}{\sigma_w} \exp \left[ -\frac{(\omega_a + \omega_i)^2}{2(\sigma_i^2 + \sigma_c^2)} \right]. \] (14)

\[ \tilde{A}_n'' = \tilde{A}_n \frac{\sigma_i'}{\sigma_w} \exp \left[ -\frac{(\omega_a + \omega_i)^2}{2(\sigma_i^2 + \sigma_c^2)} \right]. \] (15)

The right-hand-sides of eqs (8) and (9) are summations of Gaussian wavelets weighted according to their frequencies. We note that eqs (8)–(16) are exact and no approximations were involved in the derivation. To verify these equations, we give a numerical example. In eq. (4), we specify \( \tilde{A}_n = (2\pi/\delta_t) \exp[-(\omega_a - \omega_i)^2/(2\delta^2)] \), where \( \delta_t = 1.0 \times 2\pi \) and \( \delta = 0.25 \times 2\pi \). Fig. 3 shows \( \tilde{A}_n \), as a function of \( \omega_a \) and its corresponding \( \tilde{C}_{f_f}(t) \). For the windowing and narrowband filtering parameters, we choose \( \sigma_w = 0.2 \), \( \omega_i = 1.5 \times 2\pi \) and \( \sigma_c = 0.1 \times 2\pi \). In Fig. 4 we compare the \( F_iW\tilde{C}_{f_f}(t) \) obtained from the summation on the right-hand-side of eq. (8) with the one obtained by applying the time window and the narrowband filter onto \( \tilde{C}_{f_f}(t) \) numerically, they are identical up to machine precision. The same experiments were repeated for different types of \( \tilde{A}_n \) as well as different values of the windowing and filtering parameters and they all confirm the correctness of eqs (8)–(16).

We can evaluate the amplitude \( \tilde{A} \), the centre frequency \( \tilde{\omega}_1 \) and the half-bandwidth \( \tilde{\sigma}_c \) of the windowed and narrowband filtered autocorrelation from the Fourier transform of \( F_iW\tilde{C}_{f_f}(t) \).

\[ \tilde{A} = \sum_n (\tilde{A}_n + \tilde{A}_n' + \tilde{A}_n''), \] (17)

\[ \tilde{\omega}_1 = \sum_n (\tilde{A}_n \omega_a + \tilde{A}_n' \omega_a')/\tilde{A}. \] (18)
The coefficients $\omega_i$ instead of a Gaussian time window. The effect is that the autocorrelation of the isolation filter $\hat{C}_{ff}(t)$ is as less distorted by windowing as possible, although the extraneous phases are excluded from the time window. In this case, the effective bandwidth of the time window is close to zero, $\kappa_i \to 0$. For the special case, where $A_n$ is a constant, $\hat{A}_n = \hat{A}$ and $\sigma_i/\omega_i \ll 1$, we have $\hat{\omega}_i = \omega_i$, $\hat{\sigma}_i = \sigma_i$, the filtered and windowed auto-correlagram becomes an exact Gaussian wavelet,

$$F_i W \hat{C}_{ff}(t) = \hat{A} \exp(\hat{\omega}_i^2 t^2/2) \cos(\hat{\omega}_i t).$$

In general, the third- and higher-order terms in eq. (20) are non-zero but the leading term in eq. (20) still provides a good approximation to the filtered and windowed auto-correlagram as long as the variation of $\hat{A}_n$ within the effective bandwidth is sufficiently small, $\|\Delta \hat{A}\|/\hat{\sigma}_i \ll 1$, where $\|\Delta \hat{A}\|$ can be defined as the total variation of $\hat{A}_n$ within the interval $[\hat{\omega}_i - \hat{\sigma}_i, \hat{\omega}_i + \hat{\sigma}_i]$. We note that different from Gee & Jordan (1992), in this derivation, the accuracy of the Gaussian-wavelet approximation provided by the leading term in eq. (20) does not depend on the frequency-shift parameter $\xi_i$ as defined in eq. (32) in Gee & Jordan (1992) (i.e. the position of the narrowband filter $F_i$ relative to the centre frequency of the windowed auto-correlagram $W \hat{C}_{ff}$) but depends on the variation of $\hat{A}_n$ within the bandwidth $[\hat{\omega}_i - \hat{\sigma}_i, \hat{\omega}_i + \hat{\sigma}_i]$. For the special case where $\hat{A}_n$ is Gaussian (e.g. Fig. 3a), the derivative of $\hat{A}_n$ with respect to frequency is zero at centre frequency $\hat{\omega}_i$ and reach maximum at about $\hat{\omega}_i + \hat{\sigma}_i$, therefore the accuracy of the approximation provided by the leading term is higher at $\hat{\omega}_i$ than at $\hat{\omega}_i + \hat{\sigma}_i$ (Fig. 5). One implication of this derivation is that a good approximation can always be enforced by selecting a sufficiently small $\sigma_i$ so that eq. (22) is satisfied. In Fig. 5, we compare the $F_i W \hat{C}_{ff}$ computed using the right-hand-side of eq. (8) with the one provided by the leading term in eq. (20). The approximation is more accurate when the narrowband filter $F_i$ is centred near the centre frequency $\hat{\omega}_i$, where the total variation $\|\Delta \hat{A}\|$ is smaller and the accuracy improves as the bandwidth of $F_i$ reduces.

In actual computation, the parameters of the Gaussian wavelet are not obtained from the theoretical values given in eqs (17)–(19). Instead, they are estimated numerically by minimizing a $\chi^2$ quadratic form as defined in eq. (7) of Gee & Jordan (1992) using a least-squares procedure. The deviations of the numerically estimated values from the theoretical ones are due to contributions from higher-order terms in eq. (20) and are negligible when eq. (22) is satisfied.
We can express the composite Gaussian wavelets for the filtered, windowed correlagrams as

\[ F_i W C_{f_i}(t) \approx \tilde{g}(t) = \tilde{A} \exp \left[ -\frac{\tilde{\delta}^2(t - \tilde{t}_p)^2}{2} \right] \cos \left[ \tilde{\omega}_n(t - \tilde{t}_p) \right] \]

(23)

And the GSDF measurements of phase-delay time and amplitude-reduction time associated with filter \( F_i \) are defined as (Gee & Jordan 1992)

\[ \delta t_p = t_p - \tilde{t}_p, \]

(25)

\[ \delta t_q = -\frac{\ln(A/\tilde{A})}{\tilde{\omega}_n}. \]

(26)

### 4.2 Perturbation formulae

The observables, \( \delta t_{p,q} \), which are our GSDFs, can be related back to \( \delta \tau_{p,q} \) by applying the perturbation formulae, eqs (84) and (85) in Gee & Jordan (1992). We first define four parameters following eqs (73) and (74) in Gee & Jordan (1992),

\[ \tilde{B}'_p = \frac{\tilde{A}'}{A} \exp \left[ -\frac{(\sigma_p^2 t_p^2)}{2} \right], \quad \delta \tau_{p} = (\omega_n^p \tilde{t}_p^p - \tilde{\omega}_n \tilde{t}_p) + (\tilde{\omega}_n - \omega_n^p) \tilde{t}_p. \]

(29)

For the auto-correlagram, we have \( \tilde{\tau}_p^p = \tilde{\tau}_q^q = 0 \), thus we have \( \tilde{t}_p = \tilde{t}_q = 0 \) and the four parameters become

\[ B'_p = \frac{A'}{A}, \quad B''_p = \frac{A''}{A}, \quad \delta \tau_p = \delta \tau_q = 0. \]

(30)

Following eqs (75) and (76) in Gee & Jordan (1992), we define two \( 1 \times 2N \) dimensionless vectors \( \mathbf{C} \) and \( \mathbf{S} \) with their elements given by

\[ \mathbf{C} = [B'_p \cos(\varphi'_p), \ B''_p \cos(\varphi''_p)], \]

(31)

\[ \mathbf{S} = [B'_p \sin(\varphi'_p), \ B''_p \sin(\varphi''_p)]. \]

(32)

Following eqs (84) and (85) in Gee & Jordan (1992), the observables, \( \delta t_{p,q} \), can then be expressed as

\[ \delta t_p = \mathbf{C} \mathbf{t}_p - \mathbf{S} \mathbf{t}_q, \]

(33)

\[ \delta t_q = \mathbf{C} \mathbf{t}_q + \mathbf{S} \mathbf{t}_p, \]

(34)
Figure 5. Comparison between the $F_iW\tilde{C}_f(t)$ provided by the leading term of eq. (20) (thick dash lines in panels a, b, e and f) and those obtained from the right-hand-side of eq. (8) (thin solid lines in panels a, b, e and f). Panels c, d, g and h show the differences between the thick dash line and the thin solid line in panels a, b, e and f. For panels a, b, c and d, the narrowband filter $F_i$ has a half-width of 0.2 Hz. For panels e, f, g and h, $F_i$ has a half-width of 0.1 Hz. For panels a, c, e and g, $F_i$ has a centre frequency of 1.0 Hz. For panels b, d, f and h, $F_i$ has a centre frequency of 1.5 Hz. The total variance $\|\Delta A\|$ is 0.0236 for panels a and c and 0.0897 for panels b and d. The centre frequency of $F_iW\tilde{C}_f(t)$, $\tilde{\omega}_1/(2\pi)$, is 1.0 Hz for panels a, c, e and g, 1.31 Hz for panels b, d and 1.43 Hz for panels f, h. The effective bandwidth, $\sigma_1/(2\pi)$, is 0.16 Hz for panels a, b, c and d, 0.093 Hz for panels e, f, g and h. This figure demonstrates that the approximation provided by the leading term of eq. (20) has higher accuracy, where $\|\Delta A\|$ is smaller and the approximation error can be reduced by reducing the bandwidth of the narrowband filter $F_i$. 

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where \( \delta_{p,q} \) are two \( 2N \times 1 \) vectors with their elements given by

\[
\delta_{p} = \begin{bmatrix} \tau_p^n - \tilde{\tau}_p^n \tau_p^m - \tilde{\tau}_p^m \end{bmatrix} = \begin{bmatrix} \omega_0 - \omega_0' \tau_p^n + \omega_0' \tau_p^m \end{bmatrix},
\]

(35)

\[
\delta_{q} = \begin{bmatrix} -\ln(A_q^n) / |\omega_q^n| \end{bmatrix} = \begin{bmatrix} \omega_0 \delta_{q}^{n} \end{bmatrix}.
\]

(36)

Considering eq. (30), we have \( S = 0 \) and the perturbation eqs (33) and (34) are fully decoupled

\[
\delta_{p} = C \delta_{t},
\]

(37)

\[
\delta_{q} = C \delta_{t},
\]

(38)

and we can express the observables, \( \delta_{p,q} \), using \( \delta_{p,q} \) as

\[
\delta_{p} = \frac{\omega_0}{A \omega_0} \sum_n \frac{\omega_0 A_n}{\omega_0} \exp \left[ -\frac{(\omega_0 - \omega_0')^2}{2 (\sigma_0^2 + \sigma_0'^2)} \right] \delta_{t}^{n} + \frac{\omega_0}{A \omega_0} \sum_n \frac{\omega_0 A_n}{\omega_0} \exp \left[ -\frac{(\omega_0 - \omega_0')^2}{2 (\sigma_0^2 + \sigma_0'^2)} \right] \delta_{t}^{n},
\]

(39)

\[
\delta_{q} = \frac{\omega_0}{A \omega_0} \sum_n \frac{\omega_0 A_n}{\omega_0} \exp \left[ -\frac{(\omega_0 - \omega_0')^2}{2 (\sigma_0^2 + \sigma_0'^2)} \right] \delta_{t}^{n} + \frac{\omega_0}{A \omega_0} \sum_n \frac{\omega_0 A_n}{\omega_0} \exp \left[ -\frac{(\omega_0 - \omega_0')^2}{2 (\sigma_0^2 + \sigma_0'^2)} \right] \delta_{t}^{n}. \]

(40)

When \( \sigma_c \ll \omega_0 \), the second summations in eqs (39) and (40) become negligible and in the limit \( \sigma_c \to 0 \), we have

\[
\delta_{p,q} = \frac{1}{A} \sum_n \frac{\omega_0 A_n}{\omega_0} \exp \left[ -\frac{(\omega_0 - \omega_0')^2}{2 \sigma_0^2} \right] \delta_{t}^{n}.
\]

(41)

The observables, \( \delta_{p,q} \), can therefore be expressed as weighted summations of \( \delta_{p,q} \) with the weights determined by the product of the narrowband filter \( F_{t} \) and \( A_{n} \), which is the spectrum of the autocorrelation \( \Gamma_{ff} \). Writing in continuous form, we have

\[
\delta_{p,q} = \frac{1}{A} \int d\omega \left[ \tilde{\Gamma}_{ff}(\omega) \exp \left[ -\frac{(\omega - \omega_0')^2}{2 \sigma_0^2} \right] \delta_{t,q}^{n} \right].
\]

(42)

We note that, different from eqs (57) and (59) in Gee & Jordan (1992), the derivation given here is not based on a Taylor expansion of the differential propagation operator [i.e. eq. (44) in Gee & Jordan (1992)] about the centre frequency of the narrowband filtered, windowed auto-correlation and eqs (39)-(42) hold even when the frequency-shift parameter \( \xi_{1} \) as defined in eq. (32) in Gee & Jordan (1992) is large. To demonstrate this difference, let's consider the special case where \( \tilde{\Gamma}_{ff}(\omega) \) is Gaussian with centre frequency \( \omega_{c} \) and half-width \( \sigma_{c} \),

\[
\tilde{\Gamma}_{ff}(\omega) = \frac{\sqrt{2\pi}}{\sigma_c} \exp \left[ -\frac{(\omega - \omega_c)^2}{2 \sigma_c^2} \right].
\]

(43)

In this case, the weighting function in eq. (42) is also Gaussian and has the form

\[
V' (\omega) = \frac{1}{z} \exp \left[ -\frac{(\omega - \omega_c)^2}{2 \sigma_c^2} \right].
\]

(44)

where the centre frequency \( \omega_{c} \) and half-width \( \sigma_{c} \) are

\[
\sigma_{c}^2 = (\sigma_{t}^2 + \sigma_{c}^{-2})^{-1},
\]

(45)

\[
\omega_{c} = \frac{\sigma_{t}^2 \omega_0 + \sigma_{c}^{-2} \tilde{\omega}_c}{\sigma_{t}^2 + \sigma_{c}^{-2}}
\]

(46)

and \( z \) is a normalization factor. We expand \( \delta_{p,q}(\omega) \) in a Taylor series about the centre frequency \( \omega_{c} \),

\[
\delta_{p,q}(\omega) = \delta_{p,q}(\omega_{c}) + (\omega - \omega_{c}) \delta_{p,q}(\omega_{c}) + \frac{1}{2} (\omega - \omega_{c})^2 \delta_{p,q}(\omega_{c}) + \cdots
\]

(47)

and bring eq. (47) into eq. (42), we obtain

\[
\delta_{p,q} = \delta_{p,q}(\omega_{c}) + \sigma_{c}^2 \frac{\sigma_{c}^{-2}}{2 \sigma_{c}^{-2}} \delta_{p,q}(\omega_{c}) + \cdots
\]

(48)

In the limit \( \sigma_{c} \to 0 \), the time-localization control parameter, eq. (26) in Gee & Jordan (1992), \( \xi_{1} \to 1 \) and the centre frequency and half-width as defined in eqs (35) and (36) in Gee & Jordan (1992), \( \tilde{\omega}_c \to \omega_{c} \), \( \sigma_{c} \to \sigma_{c} \). In this case, eqs (57) and (59) in Gee & Jordan (1992) become

\[
\delta_{p,q} = \delta_{p,q}(\omega_{c}).
\]

(49)

Comparing eq. (49) with eq. (48), the first correction term is proportional to \( \delta_{p,q}(\omega_{c}) \). Eq. (49) is valid, while the effective bandwidth \( \sigma_{c} \) is sufficiently small and the linear approximation is adequate.

The derivation above is under the assumption that \( \sigma_{c} \to 0 \). If we assume the limit \( \sigma_{c} \to 0 \) instead, then we have \( \delta_{p,q} \to \tilde{\delta}_{p,q} \). If we expand \( \delta_{p,q}(\omega) \) in Taylor series about the frequency \( \tilde{\omega}_c \),

\[
\delta_{p,q}(\omega) = \delta_{p,q}(\tilde{\omega}_c) + (\omega - \tilde{\omega}_c) \delta_{p,q}(\tilde{\omega}_c) + \frac{1}{2} (\omega - \tilde{\omega}_c)^2 \delta_{p,q}(\tilde{\omega}_c) + \cdots
\]

(50)

bring the expansions into the continuous forms of eqs(39) and (40), ignore contributions from the second summations and assume that \( \omega_0/\omega_c \approx 1 \) within the effective bandwidth, we obtain

\[
\delta_{p} = \delta_{p} (\tilde{\omega}_c) - (1 - \xi_{1}^2) \frac{\tilde{\delta}_{p} (\tilde{\omega}_c)}{\tilde{\omega}_c} \delta_{t}(\tilde{\omega}_c) + \frac{\tilde{\delta}_{p} (1 - \xi_{1}^2)}{2} \frac{\tilde{\delta}_{t} (\tilde{\omega}_c)}{\tilde{\omega}_c} \delta_{t}(\tilde{\omega}_c),
\]

(51)

\[
\delta_{q} = \delta_{q} (\tilde{\omega}_c) - (1 - \xi_{1}^2) \frac{\tilde{\delta}_{q} (\tilde{\omega}_c)}{\tilde{\omega}_c} \delta_{t}(\tilde{\omega}_c) + \frac{\tilde{\delta}_{q} (1 - \xi_{1}^2)}{2} \frac{\tilde{\delta}_{t} (\tilde{\omega}_c)}{\tilde{\omega}_c} \delta_{t}(\tilde{\omega}_c).
\]

(52)

In this case, the 0th- and 1st-order terms in eqs (51) and (52) have recovered eqs (57) and (59) in Gee & Jordan (1992). The 1st- and higher-order correction terms become significant when the fractional shift in centre frequency due to filtering \( (\tilde{\omega}_c - \tilde{\omega}_c)/\tilde{\omega}_c \) becomes large or quadratic dispersion is significant.

4.3 Seismogram perturbation kernel

In frequency domain, the autocorrelation of the isolation filter can be expressed as:

\[
\tilde{\Gamma}_{ff}(\omega) = \tilde{f}^{*}(\omega) \tilde{f}(\omega)
\]

(53)
and the observed waveform can be expressed as a perturbation from the
isolation filter as
\[
f(\omega) = \tilde{f}(\omega) + \delta f(\omega),
\]
where \(\delta f(\omega)\) is the instrument-filtered displacement perturbation,
that is, the frequency-domain displacement perturbation \(\delta \iota_0(x, \omega)\),
as used in eq. (2), multiplied with the instrument response. Thus the
cross-correlation \(C_{ff}(\omega)\) can be expressed as
\[
C_{ff}(\omega) = \frac{\tilde{f}^*(\omega) f(\omega)}{\tilde{f}(\omega) + \delta \iota_0(\omega)}.
\]

Following eq. (44) in Gee & Jordan (1992), we define the differential
propagation operator
\[
D(\omega) = \exp\{i\omega \delta \tau_p(\omega) - \omega \delta \tau_q(\omega)\}
\]
and we have
\[
C_{ff}(\omega) = D(\omega) \tilde{C}_{ff}(\omega).
\]
The exact Fréchet derivative of \(\tau_p(\omega)\) with respect to the waveform
\(f(\omega)\) can therefore be obtained by first linearizing the differential
propagation operator
\[
\exp\{i\omega \delta \tau_p(\omega) - \omega \delta \tau_q(\omega)\} = 1 + i\omega \delta \tau_p(\omega) - \omega \delta \tau_q(\omega) + \cdots
\]
and then considering eq. (55), we can obtain
\[
\delta \tau_p(\omega) = -\frac{i}{\omega} \text{Im} \left[ \frac{\tilde{f}^*(\omega) \delta f(\omega)}{\tilde{f}(\omega)} \right].
\]
\[
\delta \tau_q(\omega) = -\frac{1}{\omega} \text{Re} \left[ \frac{\tilde{f}^*(\omega) \delta f(\omega)}{\tilde{f}(\omega)} \right].
\]

Bring eqs (59) and (60) into the continuous forms of eqs (39)–(42),
transform \(\delta \iota_0\) into the time domain and define a function \(I(t)\) as
\[
I(t) = \int_{-\infty}^{\infty} d\omega \exp(i\omega t) \left\{ H(\omega) \frac{\sigma_p(\omega)}{\omega} \exp\left[ \frac{-(\omega - \omega_0)^2}{2(\sigma_p^2 + \sigma_e^2)} \right] \tilde{f}(\omega) \right\}
+ \int_{-\infty}^{\infty} d\omega \exp(i\omega t) \left\{ H(\omega) \frac{\sigma_q(\omega)}{\omega} \exp\left[ \frac{-(\omega + \omega_0)^2}{2(\sigma_e^2 + \sigma_q^2)} \right] \tilde{f}(\omega) \right\},
\]
where \(H(\omega)\) is the Heaviside function and
\[
\omega_0 = \frac{\sigma_p^2 \omega + \sigma_e^2 \omega_0}{\sigma_p^2 + \sigma_e^2}, \quad \omega_0' = \frac{\sigma_p^2 \omega - \sigma_e^2 \omega_0}{\sigma_p^2 + \sigma_e^2}.
\]

then the exact Fréchet derivatives of the observables, \(\delta \iota_{p,q}\), with
respect to the time-domain waveform \(f(t)\) can be expressed as
\[
\delta \iota_k = \int dt J_k(t) \delta f(t), \ (x = p, q),
\]
where the seismogram perturbation kernel \(J_k(t)\) is given by
\[
J_p(t) = \text{Im} \left[ I(t) \right],
\]
\[
J_q(t) = -\text{Re} \left[ I(t) \right].
\]
Here we have adopted the temporal Fourier transform convention
defined in Aki & Richards (2002). When the effective bandwidth
\(\sqrt{\sigma_p^2 + \sigma_e^2} \ll \omega_0\), contribution from the second integral in eq. (64)
becomes negligible. And in the limit \(\sigma_p \rightarrow 0\), we have
\[
I(t) = \frac{1}{A} \int_{-\infty}^{\infty} d\omega \exp(i\omega t) \left\{ H(\omega) \exp\left[ \frac{-(\omega - \omega_0)^2}{2\sigma_e^2} \right] \tilde{f}(\omega) \right\},
\]
In this case, the seismogram perturbation kernel is related to the
narrowband-filtered synthetic velocity at the receiver.

We note that the instrument response, as well as any pre-filtering
operators, in \(\delta f(t)\) can also be absorbed by \(J_k(t)\), in which case
eq. (63) can be converted into a linear relation between \(\delta \iota_k\) and
the time-domain displacement perturbation \(\delta \iota_0(x, t)\) in the form of eq.
(2). We also note that in the adjoint-wavefield method for F3DT,
the adjoint source field (Tarantola 1986; Akcelik et al. 2003; Tromp
et al. 2005; Liu & Tromp 2006) is given by the time-reversed \(J_k(t)\)
weighted by its corresponding GSDF observable \(\delta \iota_k\) and located
at its receiver location (Chen et al. 2007a). An example of \(J_k(t)\), as well
as its spectrum, is shown in Fig. 6. The width, as well as the
oscillatory character, of the corresponding data sensitivity kernels is
controlled by the effective bandwidth and the narrowband filtering
frequency \(\omega_0\) as demonstrated in Figs 7–9.

4.4 Noise model of the GSDF measurements
Let \(\Delta f(t) = \delta f(t) + n(t)\), where \(n(t)\) is a stationary noise process
with zero mean and autocovariance \(V_n(t) = V_n(-t)\):
\[
\langle n(t) \rangle = 0, \quad \langle n(t)n(t') \rangle = V_n(|t - t'|),
\]
where \(\langle \rangle\) denotes averaging through time \(t\). The noise power spectrum is
\[
P_n(\omega) = \int_{-\infty}^{\infty} V_n(t) \exp(i\omega t) dt.
\]
Then we have
\[
\langle \Delta \iota_k \rangle = \int_{-\infty}^{\infty} J_k(t) \Delta f(t) dt = \delta \iota_k, \quad \langle |\Delta \iota_k - \delta \iota_k|^2 \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J_k(t) J_k(t') V_n(t - t') dt dt',
\]
\[
= \frac{1}{\pi} \int_0^{\infty} |J_k(\omega)|^2 P_n(\omega) d\omega.
\]

In general, the statistical properties of the noise, in particular the
‘signal-generated’ noise due to inadequate modelling assumptions,
are not known. However, as demonstrated in Chen et al. (2007b),
simple noise models can be constructed based on sampling frequencies
\(\omega_0\) and the types of the wave groups used for making GSDF
measurements.

5 Numerical Examples

5.1 Fréchet kernels in a uniform half-space
Following Zhao et al. (2005), for our first numerical example, we use
a simple half-space model with a constant density \(\rho = 3000 \text{ kg/m}^3\)
and \(P\)-wave speed \(\alpha = 6.5 \text{ km/s}\) and \(S\)-wave speed \(\beta = 3.5 \text{ km/s}\).
The seismic source and the receiver are both buried at 24 km depth
and the source-receiver distance is 32.2 km. We use an explosive
source with the source-time function given by
\[
s(t) = \exp[-a(t - h/2)^2].
\]
Figure 6. Seismogram perturbation kernels $J_q(t)$ (left-hand column) and $J_p(t)$ (right-hand column) for the isolation filter used in Fig. 8 at five different narrowband filtering frequencies from 0.5 Hz to 2.5 Hz. The amplitude spectrums of $J_q(t)$ at the five frequencies are shown in the middle column. The amplitude spectrums of $J_p(t)$ are similar to those of $J_q(t)$.

Here we choose $a = 60$ and $b = 0.65$, resulting in synthetics with maximum frequency of about 2 Hz. Fig. 7 shows the Fréchet kernels for the frequency-dependent GSDF phase-delay and amplitude-reduction times for the three arrivals on the radial component of the buried receiver: the direct-arriving $P$ wave, the free surface reflected $pP$ and $pS$ waves. In this uniform half-space model, the $P$-wave ray path is the straight line that connects the source and the receiver. The sensitivity for the $P$-wave is not concentrated on the ray path but extends to as far as 8 km away from the ray path. For the kernels of the frequency-dependent phase-delay times, we also see the ‘banana-doughnut’ phenomena of vanishing sensitivity on the ray path (Marquering et al. 1999; Dahlen et al. 2000; Hung et al. 2000; Zhao et al. 2000; Zhao et al. 2005). The width of the kernel is generally controlled by the effective bandwidth of the narrowband filtered, windowed auto-correlagram, $\tilde{\sigma}$, and the width of the first Fresnel zone and the spatial oscillation of the sensitivity depends on the frequency $\tilde{\omega}$. The sensitivity kernels for the surface-reflected phases $pP$ and $pS$ have similar characteristics as the direct-arriving $P$-wave. However, the width of the first Fresnel zone for these surface-reflected phases is larger than the direct-arriving $P$-wave due to longer propagation distances than the $P$-wave.

5.2 Fréchet kernels for ambient-noise Green’s functions

Recently, there has been increased interest in using Green’s functions extracted from ambient-noise recordings to estimate earth structures. We would like to point out the possibility of applying GSDF analysis on ambient-noise Green’s function data. In this section, we give two numerical examples of Fréchet kernels of GSDFs measured on ambient-noise Green’s functions using a uniform half-space model and a complex 3-D structure model.

5.2.1 Background and motivation

Theoretical and experimental studies (e.g. Lobkis & Weaver 2001; Weaver & Lobkis 2001, 2004; Snieder 2004; Wapenaar 2004; Sánchez-Sesma & Campillo 2006; Sánchez-Sesma et al. 2006) have shown that stacking cross-correlations of noise recordings at two stations produces an estimate of the Green’s function of the material recorded at one station as if an impulse excitation was acting on the other station. For a dense seismic network, ambient seismic noise data can take advantage of the density of the station coverage and do not require earthquakes or active sources. So far, ambient-noise Green’s function data have been used in a number of tomographic studies in different areas to obtain group and/or phase velocity maps at different periods (e.g. Shapiro et al. 2005; Gerstoft et al. 2006; Yao et al. 2006; Brenguier et al. 2007; Cho et al. 2007; Lin et al. 2007; Yang et al. 2007; Bensen et al. 2008).

Ambient-noise Green’s function data can also provide an independent way to estimate the quality of 3-D seismic structure models (Ma et al. 2008). Recently, Ma et al. (2008) compared the observed and synthetic ambient-noise Green’s functions between stations in Southern California to evaluate two 3-D seismic velocity models, the Southern California Earthquake Center (SCEC) Community Velocity Model Version 4.0 (CVM 4.0; Kohler et al. 2003; Magistrale et al. 2000) and a model that integrates reflection/refraction data from the industry (Süss & Shaw 2003), CVM-H 5.2. The synthetic Green’s functions were computed using a finite-element method (Ma & Liu 2006) with unit impulsive force acting at a station location. The observed and synthetic Green’s functions...
were bandpass filtered between 0.1 and 0.2 Hz, which is the frequency band that contains the dominant energy in the observed ambient-noise Green’s functions (Ma et al. 2008). The results show that the synthetic Green’s functions generated using CVM 4.0 have higher waveform similarity with observed ambient-noise Green’s functions, although those generated using CVM-H 5.2 provide better fit to the observed surface-wave arrival-times (Ma et al. 2008). Based on those evaluation results, both 3-D velocity models, CVM 4.0 and CVM-H 5.2, can be used as suitable starting models in F3DT.

### 5.2.2 F3DT using ambient-noise Green’s functions

To adapt F3DT to use ambient-noise Green’s function data, we need to quantify the differences between synthetic and observed Green’s functions and compute Fréchet kernels for the misfit measurements. The synthetic Green’s functions for a station pair can be generated by applying a unit impulsive force on one ‘source’ station and recording the waveform on the other ‘receiver’ station. We can then apply the GSDF waveform analysis procedure to extract frequency-dependent phase-delay times and amplitude-reduction times to quantify waveform differences.

So far, both the adjoint-wavefield method (Tromp et al. 2005; Liu & Tromp 2006) and the SI method (Zhao et al. 2005) have been adopted to calculate full 3-D Fréchet kernels. In the adjoint-wavefield method the time-reversed seismogram perturbation kernels located at receivers and weighted by the misfit measurements are used as the adjoint source field in adjoint simulations to construct the gradients of the objective function (Liu & Tromp 2006; Tape et al. 2007). In the SI method, the sensitivity kernels for individual misfit measurements are constructed by convolving the receiver-side Green’s tensor, which is the wavefields generated by the three orthogonal unit impulsive point forces acting at the receiver location, with the forward wavefield generated by the source (Zhao et al. 2005) and then integrate against the seismogram.
p perturbation kernel. Following the notation in Chen et al. (2007b), the sensitivity kernel of the GSDF measurements with respect to the shear-wave speed can be expressed as:

\[ K_{\beta}^{\text{sr}}(x) = -2\rho(x)\beta(x) \int dt J_{\text{sr}}^{\text{in}}(t) \int d\tau \left\{ 2 \sum_{j,l} \left[ \partial_j G(x, t - \tau; x_r) \times \partial_l u(x, \tau) \right] - \sum_{j,k} \left[ \partial_j G(x, t - \tau; x_r) \left[ \partial_j u(x, \tau) + \partial_k u(x, \tau) \right] \right] \right\}, \]  

where the seismogram perturbation kernel \( J_{\text{sr}}^{\text{in}}(t) \) is given in eqs (61)–(65). To construct the sensitivity kernels for GSDFs measured on ambient-noise Green's functions between a ‘receiver’ stations \( r_1 \) and a ‘source’ station \( r_2 \), in eq. (73) the receiver Green tensor becomes

\[ G(x, t; x_1) = G(x, t; x_1) \]  

and the forward wavefield from the source is

\[ u(x, t) = G(x, t; x_2) \cdot \hat{e}, \]  

where \( \hat{e} \) is a polarization vector determined by the direction of the impulsive force acting at the ‘source’ station \( r_2 \). With these modifications, eq. (73) provides the foundation for computing Fréchet kernels for ambient-noise Green's functions based on the SI method.

5.2.3 Kernel examples

In our first kernel example for ambient-noise Green's functions, we solve the Lamb's problem (Lamb 1904) in a homogeneous half-space model, with constant P-wave velocity (6500 m/sec), S-wave

Figure 8. Fréchet kernels for a Rayleigh wave from a vertical point impulsive force acting at station STA2 and recorded at station STA1. Both stations are on the free surface. A uniform half-space structure model was used in the calculation. The vertical-component synthetic seismogram is shown in panel a, the isolation filter is highlighted in red. Panels b and c show the seismogram perturbation kernels for the GSDF phase-delay time \( \delta t_p \) at 1.0–2.5 Hz. Panels d and e show the map-views of the Fréchet kernels at 1.0 km depth for \( \delta t_p \) at 1.0 Hz (panel d) and 2.5 Hz (panel e) with respect to S-wave speed \( \beta \). Panels f and g show the cross-section views of the Fréchet kernels for phase-delay times with respect to \( \beta \) at the station–station vertical plane.
The seismogram perturbation kernels for GSDF phase-delay times at 1.0–2.5 Hz are shown in Figs 8(b) and (c). The $\beta$ sensitivity kernels at 1.0–2.5 Hz are shown in Figs 8(d)–(g). From the map-views and cross-sections of the $\beta$ sensitivity kernels we can see that the kernel at 1.0 Hz has a wider first Fresnel zone and higher sensitivity at larger depth than the kernel at 2.5 Hz. At lower frequencies, the wavelength is longer, thus the seismic wave is sensitive to a larger area off the ray path.
In our second example, we adopted the 3-D SCEC CVM 4.0 as our structure model and used two broad-band stations SLA and FMP for computing synthetic Green’s functions and their sensitivity kernels. The SLA station is located near the Death Valley National Park and the FMP station is located in Fort Macarthur Park and the distance between the two stations is about 200 km. We applied a vertical unit impulse force on ‘source’ station FMP and station SLA acted as a ‘receiver’ station (Fig. 9). The dispersion effect is visible on the vertical-component synthetic seismogram (Fig. 9a). The seismogram perturbation kernels for GSDF phase-delay times at 0.05–0.15 Hz are shown in Figs 9(b) and (c). The map-views and cross-sections of delay times at 0.05–0.15 Hz are shown in Figs 9(d)–(g). We also observe frequency-dependence of the kernels. The complexity of the kernels is due to the 3-D heterogeneities in the structure model.

6 DISCUSSION

The formulation given in this paper departs from that given in Gee & Jordan (1992) at eq. (4), in which we expand the autocorrelation \( \tilde{C}_{ff}(t) \) into a Fourier series instead of a Gram-Charlier series as given by eq. (15) in Gee & Jordan (1992). The consequence of this modification is two-folded. First, it allows us to remove the restriction on the frequency-shift parameter \( \xi \) and replace it with a much less restrictive requirement, that is, as long as the variation of \( \tilde{A} \) within the effective bandwidth is sufficiently small, the windowed and narrowband-filtered auto-correlagram \( F \tilde{W} \tilde{C}_{ff}(t) \) can always be well matched by a five-parameter Gaussian wavelet. Secondly, it allows us to avoid a Taylor expansion of the differential propagation operator at a reference frequency as given by eq. (44) in Gee & Jordan (1992) and 2nd- and higher-order dispersion effects are automatically accounted for in eqs (39)–(42) and (61)–(66). In Gee & Jordan (1992), the sensitivity kernels are computed for \( \delta x \) as shown in eq. (93) and \( \delta x \) are obtained from the GSDF observables \( \delta x \) by solving eqs (56)–(59) (i.e. correcting the effects of windowing and narrowband filtering on the target waveform), which might become unstable when \( |\xi| \) is large. In this paper, we directly construct the sensitivity kernels for the GSDF observables \( \delta x \) and account for the effects of windowing and narrowband filtering in the seismogram perturbation kernels \( J_i(t) \), which improves the stability of the measurements and the kernels. We also gave a noise model for the GSDF observables \( \delta x \) based on the seismogram perturbation kernels and a noise power spectrum.

The GSDF analysis has been successfully applied to image seismic velocity structures using waveform data from natural earthquakes (e.g. Gaherty et al. 1996; Katzman et al. 1998; Chen et al. 2007b) as well as to inversions for earthquake source parameters (e.g. McGuire et al. 2001; Chen et al. 2005; Chen et al. 2010). The same methodology can also be applied on ambient-noise Green’s function data, which can be easily incorporated into full-3D tomographic inversions and potentially provide additional constraints on earth structures.

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