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Economics of Fertilizer Use and Analysis of Nitrogen Use on Barley

By G. P. Roehrkasse
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Economics of Fertilizer Use and Analysis of Nitrogen Use on Barley

By Glenn P. Roehrkasse

The increased use of commercial fertilizers by Wyoming farmers since 1940 has been one of the important developments in Wyoming agriculture. From 1943 through 1959 the use of commercial fertilizer by Wyoming farmers increased at an average rate of 728 tons per year. This increased usage is an average of approximately 8.99 percent per year for the 1943-1959 period (Fig. 1). That trend is still upward. Consequently, commercial fertilizers have become a major variable productive resource used in many sectors of Wyoming agriculture. As such, economic guides for fertilizer use are essential for efficient agriculture.

Often recommendations as to fertilizer use have been based on experimental results involving too few levels of fertilizer application and with very little, if any, attention to the economic factors such as fertilizer and product price, the capital position of the farmers, and risk and uncertainty. In order to improve and provide more nearly complete recommendations, more fertilizer studies are needed which provide information on the many facets of productivity and also permit an economic analysis of fertilizer use.

FIG. 1—Annual tonnage of commercial fertilizer used in Wyoming, 1930-1959.

The secular-trend equation that was estimated from the 1943-1959 data on the tons of commercial fertilizers used is \( Y = 8,101.29 + 728.02x \), where \( Y \) denotes the tons of commercial fertilizer used and \( x \) is an index of time measured as deviations from the year 1951. The value 8,101.29 is the mean number of tons of commercial fertilizer during the 1943-1959 period.

/ Associate Professor of Agricultural Economics, University of Wyoming.

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OBJECTIVES

A knowledge of the fertilizer crop response function is basic to an economic analysis of fertilizer use. For farmers to allocate fertilizer resources optimally within their business, they need information as to the expected incremental yields forthcoming from different levels of intensity of fertilizer use. The yield of a particular crop is a function of many factors such as climate, soil factors, variety, and management practices. Since these factors are not generally known or measurable, there exists no unique function which will fit all situations. However, it is practical to make separate investigations where differences do exist due to factors such as climatic conditions and or soil types.

The primary objective of this study is to present the basic logic underlying agronomic and economic relationships of fertilizer use. Many of the basic principles presented in this study will be illustrated with an empirical analysis of a barley nitrogen experiment. The emphasis, in this first of a series of agronomic economic studies, is placed on the basic relationships and principles in the hope that these basic relationships and principles will serve as a guide for future fertilizer crop research in order that they may be better adapted to economic analysis.

BASIC CONCEPTS IN ECONOMIC ANALYSIS OF FERTILIZER USE

To provide the basis for formulating the logic that will be adapted to fertilizer-use problems, review of the relevant economic theory and production relationships seems necessary. These fundamental concepts will be presented, not necessarily in detail but rather to show the relationships that can be adapted so as to provide economic analysis of fertilizer use. These concepts so adapted will provide the basis for the hypotheses and the theoretical framework within which this study is conducted.

Fertilizer Crop Response Functions

Crop production is a complex process whereby many different resources (called inputs) are combined together and transformed into some specific crop (called the output). The relationship whereby resources are transformed into a crop can be characterized by a crop-production function. Such a function is a mathematical expression describing the functional relationship between the resource inputs and the crop output.

Knowledge of the production function is of primary importance, for not only does it show the input output transformation; it also serves as the basis for the derivation of necessary physical quantities, which when combined with
price data provide information needed for choice and decision making.

The crop-production function in its most general form may be written as:

\[ C = f(F_1, F_2, F_3, \ldots, F_n, X_1, X_2, \ldots, X_n) \]

which implies that crop production \( C \) is a function of the fertilizer nutrients \( F_1 \) through \( F_n \) and other types of resource inputs \( X_1 \) through \( X_n \). These other resource inputs \( (X_1, X_2, \ldots, X_n) \) may include such resource categories as soil type, seed variety, seeding rate, number of irrigations, cultural practices, climatic conditions, and other resource inputs which affect crop production.

In the general crop-production function, fertilizer nutrients represent but one group of the many different resources needed. A function where fertilizer nutrients are the only resource inputs which are variable, or which can be controlled and where all other resource inputs are fixed at some constant level, may be written as:

\[ C = f(F_1| F_2, F_3, \ldots, F_n, X_1, X_2, \ldots, X_n) \]

All resources to the left of the vertical bar are considered to be the variable resources, while all resources to the right are held fixed at some constant level. Hence this function expresses the relationship between crop yield and varying rates of the fertilizer nutrients \( F_1 \) through \( F_n \), while all the other resource inputs are held constant at some predetermined level.

While this latter function assumes that all the fertilizer nutrients are variable-resource inputs, most fertilizer studies are conducted under the assumption that either one, two, or three fertilizer nutrients are variable-resource inputs.

**Fertilizer crop response function with a single-variable nutrient**

The simplest fertilizer crop response function is a function where only one fertilizer nutrient is variable and where all other nutrients as well as other resource inputs are held fixed at some pre-determined level. Such a response function would be of the general form:

\[ C = f(F_1| F_2, F_3, \ldots, F_n, X_1, X_2, \ldots, X_n) \]

However, since all factors that affect crop yields cannot be controlled, then the fertilizer crop response function would be written as:

\[ C = f(F_1| F_2, F_3, \ldots, F_n, X_1, X_2, \ldots, X_n) + u, \]

where \( u \) is an error term which is randomly and independently distributed.

Past experimental evidence as well as biological and economic logic suggests that the fertilizer crop response function for a single variable is one of the form as presented in Figure 2. Thus Figure 2 illustrates the relationship be-
between one variable nutrient (F<sub>1</sub>) and the output of the crop with all other factors held constant at some fixed level. The concave nature of the curve indicates the diminishing marginal productivity of the fertilizer nutrient F<sub>1</sub>. If ΔC refers to the change in crop yield and ΔF<sub>1</sub> refers to the change in fertilizer input of nutrient F<sub>1</sub>, then the ratio \( \frac{\Delta C}{\Delta F_1} \) is referred to as the marginal product\(^1\) of the fertilizer nutrient F<sub>1</sub>. Thus the ratio \( \frac{\Delta C}{\Delta F_1} \) serves as one of the factors in determining the most profitable level of fertilizer use.

As indicated in Figure 2, the ratio \( \frac{\Delta C}{\Delta F_1} \) diminished with increased use of the nutrient F<sub>1</sub>. The diminishing nature of the marginal physical product, with increased use of the nutrient F<sub>1</sub>, means that the total crop yield is increasing but at a decreasing rate. However, total crop yield will continue to increase only to a point beyond which any additional use of the fertilizer F<sub>1</sub> will result in a decline in the total crop yield. This concept is illustrated in Figure 2, where A denotes the point on the fertilizer crop response function where crop yields are at a maximum. The quantity of the fertilizer F<sub>1</sub> that is needed to produce this maximum yield is OQ. Hence the quantity OQ of F<sub>1</sub> becomes the upper limit for any rational use of the fertilizer when all the other resources are held constant at some predetermined level.

The effect that varying the levels or the combination of levels at which the fixed factors are held constant may have on the relationship between crop yields and the fertilizer nutrient F<sub>1</sub> can be illustrated by considering the fertilizer crop response function when two or more factor inputs are variable.

**Fertilizer crop response function with two-or-more variable nutrients**

The fertilizer crop response function with two-or-more fertilizer nutrients variable becomes somewhat more complex than the single-variable case discussed above. For the purpose of illustration, only the case where two nutrients are considered variable, while all other factors are held constant at some predetermined level, will be discussed. However, the concepts that will be presented for the two-variable case can readily be applied to fertilizer problems involving three or more variable inputs. The fertilizer crop response function with two fertilizer nutrients variable would be of the form:

\[ \text{Fertilizer crop response function with two fertilizer nutrients variable} \]

\[ \lim_{\Delta F_1 \to 0} \frac{\Delta C}{\Delta F_1} = \frac{dC}{dF_1} \]

---

\(^1\) The marginal product computed in this manner is the average marginal product. Under the assumption that the production function is a continuous function, the exact marginal product for any given level of fertilizer use can be derived by calculus since:

\[ \lim_{\Delta F_1 \to 0} \frac{\Delta C}{\Delta F_1} = \frac{dC}{dF_1} \]
which implies that crop yields \((C)\) are a function of varying levels of the fertilizer nutrients \(F_1\) and \(F_2\), while all the remaining factors are held constant at some predetermined level.

With the aid of geometric models, it is possible to represent the fertilizer crop production function, with two nutrients variable, as a production surface. This surface represents the crop yields obtained from the use of various combinations of the two-variable fertilizer \(F_1\) and \(F_2\), while all the other factors are held constant at some predetermined level. The fertilizer crop response surface would be located graphically by measuring yields on the vertical axis, of a three-dimensional figure, and the nutrients \(F_1\) and \(F_2\) on the respective horizontal axis. This concept is illustrated in Figure 3.

Even though the production surface in Figure 3 is hypothetical, it does represent one extreme possibility in fertilizer-nutrient substitutability. This production surface assumes that the nutrients are *perfect substitutes*. That is, for the production of any given level of crop yield, one nutrient will substitute for the other nutrient at a fixed rate regardless of the proportions in which they are combined. Moreover, this production surface assumes that, if either of the nutrients are increased alone or in constant proportion, total crop yields will increase at a decreasing rate, then eventually decline. If a vertical slice is made through the production surface so as to pass through the origin, then the three-dimensional surface may be reduced to a two-dimensional diagram to
reveal a vertical profile of a response function similar in nature to the function presented in Figure 2.

On the production surface in Figure 3, the lines aa', bb', cc', and dd' are crop-yield contours depicting increasing levels of yields. The yields corresponding to the above contours are indicated as 20, 40, 60, and 80, respectively. The crop-yield contours on the production surface in Figure 3 can be reproduced two-dimensionally as shown in Figure 4. These contours have the same connotation, when they are reproduced on a two-dimensional diagram from a three-dimensional figure, as elevation contours on a topographic map. Each crop-yield contour, as shown in Figure 4, denotes a given fixed level of yield, and each point on the contour specifies the quantity of each nutrient which is required to produce this given level.

![Diagram of crop-yield contours](image)

**FIG. 4—**The crop-yield isoquant map corresponding to Figure 3, showing perfect substitution between two fertilizer nutrients.
Hence a crop-yield contour, which shows all the possible combinations of the two fertilizer nutrients that will produce a given level of yield, is called an isoquant (indicating an equal quantity of crop yield).

The crop-yield contour or isoquant bb’ in Figure 4 indicates that a yield of 40 can be produced with either (1) Ob’ of the fertilizer nutrient F₁ and none of the fertilizer F₂, or (2) Ob of the fertilizer nutrient F₂ and none of the nutrient F₁, or (3) any combination of the nutrients F₁ and F₂ as prescribed by the isoquant line bb’. Since the crop-yield isoquants are linear, the two fertilizer nutrients substitute for each other at a constant rate of substitution in the production of a given crop yield.

If ∆F₂ refers to the change in F₂ and ∆F₁ refers to the change in F₁, then the rate of substitution of the fertilizer nutrient F₁ for the fertilizer nutrient F₂ is denoted by the ratio $\frac{\Delta F_2}{\Delta F_1}$, which is also the slope of the isoquant cc’. The slope of the isoquant curve gives the rate at which one fertilizer nutrient substitutes for the other nutrient. Since a linear crop-yield isoquant has the same slope at every point, the rate of substitution between two fertilizer nutrients will always be at a constant rate. However, if the crop-yield isoquant is a curved line (non-linear), then the slope of the curve is different at every point along the curve, and the substitution rate between the two nutrients will also be different at each point. Therefore the substitution rate between any two fertilizer nutrients is technically referred to as the marginal rate of substitution, whether or not the crop-yield isoquants are linear or non-linear.

The marginal rate of substitution may be defined as the amount one fertilizer nutrient must be decreased, in order to keep the total crop yield constant, as the other fertilizer nutrient is increased by one unit.

The diminishing returns to fertilizer use, as illustrated in Figure 2 and 3, is also represented in Figure 4. When crop yield isoquants, which represent equal increments of crop yield, are successively farther apart along any straight line through the origin, diminishing returns to fertilizer use are indicated. For example, if the crop yield isoquants aa’, bb’, cc’, and dd’, in Figure 4 represent crop yields of 20, 40, 60, and 80 respectively, then for diminishing returns to fertilizer use to be evident, line segment Oa” must be less than a’b” which is less than b”c” which is less than c”d”.

The opposite extreme possibility in fertilizer-nutrient relationships is illus-
trated in Figure 5. This particular production surface assumes that the fertilizer nutrients are *perfect complements*. That is, for the production of any given level of crop yield there exists a minimum amount of the two fertilizer nutrients which will produce this given level of crop yield. Furthermore, either one, but not both, of the fertilizer nutrients may be increased beyond this minimum quantity without changing either (1) the amount required of the other fertilizer nutrient or (2) the level of crop yield.

A special case of perfect complements was advanced by soil chemists von Liebig, Meyer, and Wolney. They assumed that fertilizer nutrients must be combined in fixed proportions and that the fertilizer nutrients will not substitute one for the other in the production of a given crop yield. Moreover, if there is more or less of either of the fertilizer nutrients available than the amount required to be combined in this fixed proportion, then the crop yield becomes zero. For further details on this production function see: Heady, Earl O., Pesek, John T., and Brown, William, "Crop Response Surfaces and Economic Optima In Fertilizer Use". Iowa Agr. Expt. Sta. Res. Bul. 424. Iowa State College, Ames, Iowa, 1955.
FIG. 6—The crop-yield isoquant map corresponding to Figure 5, showing perfect complements between two fertilizer nutrients.

The isoquant map, associated with the production surface in Figure 5, is illustrated in Figure 6. The crop-yield isoquants are interpreted in the same manner as those illustrated in Figure 4. The crop-yield isoquants, as illustrated in Figure 6, indicate that the fertilizer nutrients do not substitute for each other in the production of a given level of crop yield. For a given level, the corner of the crop-yield isoquant indicates the minimum quantities of the two fertilizer nutrients that are required to produce this level. Any additional quantities of either of the fertilizer nutrients alone will have no effect on crop yield or on the amount required of the other nutrient. Since there is no substitution effect between the two fertilizer nutrients and if the fertilizer nutrients are to be applied at all, then they should always be applied in the combination indicated by the corner of the crop-yield isoquant.

The two production surfaces just discussed illustrate the two extreme cases in fertilizer-nutrient relationships: (1) the case where fertilizer nutrients are perfect substitutes and (2) the case where they are perfect complements. While these two surfaces may have some special applications, a more logical production surface for most crops and soils is illustrated in Figure 7. The concave nature of this production surface indicates diminishing returns to each fertilizer nutrient alone or to any fixed combination of the two fertilizer nutrients. The isoquant map for the production surface in Figure 7 is shown in Figure 8. The crop-yield isoquants, which are shown in both Figures 7 and 8, are curved and convex to the origin, suggesting that the fertilizer nutrients do substitute for each other at a diminishing rate in the production of a given level of crop yield. That is, for the production of any given level of crop yield, successive given amounts of either one of the fertilizer nutrients will replace in the fertilizer mix smaller and smaller amounts of the other fertilizer nutrient.

The slope of any isoquant, as mentioned previously, defines the marginal rate of substitution and is denoted by

7/ Technically, the rate of substitution between the fertilizer nutrients is either zero or infinity.
the ratio $\frac{\Delta F_2}{\Delta F_1}$. Since the yield iso-
quants, as illustrated in Figure 8, are curved, the ratio $\frac{\Delta F_2}{\Delta F_1}$ diminishes
as increasing amounts of the fertilizer nutrient $F_1$ are substituted for the nutrient $F_2$. The diminishing nature of
the $\frac{\Delta F_2}{\Delta F_1}$ indicates that the marginal

---

**FIG. 7**—A hypothetical production surface illustrating both substitution and complementarity between
two fertilizer nutrients.

*The ratio $\frac{\Delta F_2}{\Delta F_1}$ indicates only the average slope and the average marginal rate of substitu-
tion between two combinations of fertilizer use. However, the exact slope and the exact
marginal rate of substitution for any given combination of fertilizer nutrients may be derived
by calculus, since:

$$
\lim_{\Delta F_1 \to 0} \frac{\Delta F_2}{\Delta F_1} = \frac{d F_2}{d F_1}
$$
rate of substitution of nutrient $F_1$ for nutrient $F_2$ declines as increasing amounts of the nutrient $F_1$ is substituted for the nutrient $F_2$.

As illustrated in Figure 8, the lower levels of crop yield can be produced with either of the fertilizer nutrients alone as well as with some combination of the two nutrients. However, the higher levels of crop yield can be attained only with some combination of the two nutrients, which indicates some degree of complementarity between the two nutrients. Moreover, the isoquants representing higher levels of yield become shorter in length, indicating that the range of the ratio of the nutrients (the fertilizer mix) becomes increasingly narrower for successively higher levels of crop yield. Thus, the maximum crop-yield isoquant will be reduced to a single point as illustrated by the point A in Figure 8, which means that the maximum crop yield can be attained only with a single combination of the nutrients.

The line that connects points of equal slope (i.e., equal marginal rates of sub-

![Graph](image-url)

**FIG. 8**—The crop-yield isoquant map corresponding to Figure 7, illustrating both substitution and complementarity between two fertilizer nutrients.
FIG. 9—The crop-yield isoquant map corresponding to Figure 7, illustrating 4 hypothetical isoclines superimposed over the isoquants.

stitution) on successive isoquants is technically referred to as an isocline. For an isoquant map such as is illustrated in Figure 8 there exists an infinite number of isoclines. Four of these, illustrated in Figure 9, are denoted as \( I_1 \), \( I_2 \), \( I_3 \), and \( I_4 \). The isoclines, as illustrated, will converge to the point of maximum crop yield. The marginal rates of substitution corresponding to the above isoclines are indicated as 3, 2, 0.8, and 0.6, respectively. For the isocline \( I_1 \), where the marginal rate of substitution is equal to 3, it means that all along this isocline the marginal rate of substitution of nutrient \( F_1 \) for nutrient \( F_2 \) is 3. That is, one pound of fertilizer nutrient \( F_1 \) will substitute for 3 pounds of nutrient \( F_2 \). An isocline has certain economic significance and will be discussed in the next section.
Once the physical production relationships, as outlined above, have been derived, it then becomes necessary to introduce price data into the analysis in order to specify the economic optima in fertilizer use. Specifically, (1) the price of the crop being produced, and (2) the price of the fertilizer nutrients that are used in producing the crop, constitute the relevant price data that are required in specifying (1) the optimum rate of fertilizer use and (2) the optimum combination of fertilizer nutrients.

The Optimum Level of Fertilizer Use for a Single-Variable Nutrient or a Given Fertilizer Mix

In the case of a single-variable fertilizer nutrient, the optimum rate of fertilizer use is achieved under the conditions specified in equation (6) below, where the ratio $\frac{\Delta C}{\Delta F_1}$, as stated previously, is the marginal product of the fertilizer nutrient $F_1$, $\frac{P}{F_1}$ is the price per unit of the fertilizer nutrient $F_1$, and $\frac{P}{C}$ is the price per unit of the crop being produced. According to equation (6), the optimum rate of fertilizer use is attained when the marginal physical product of the fertilizer nutrient $F_1$, $\frac{\Delta C}{\Delta F_1}$, is equal to the fertilizer crop price ratio $\frac{P}{F_1}$, Equation (6a), which can be derived from equation (6), indicates that the optimum rate of fertilizer use is achieved when the value of the added crop yield $\frac{\Delta C P}{P}$ is equal to the cost of attaining this added crop yield $\frac{\Delta F_1 P}{F_1}$.

From equation (6) it is easy to see that any change in the fertilizer crop price ratio will alter the optimum rate of fertilizer use. For if the marginal physical product of the fertilizer nutrient $F_1$ is greater than the fertilizer crop price ratio as shown in equation (7) below, then the value of the added crop yield will be greater than the cost of fertilizing.

\[ \frac{\Delta C}{\Delta F_1} = \frac{F_1}{\frac{P}{C}} \]

\[ \Delta C P = \Delta F_1 P \]

The optimum rate of fertilizer use as specified by the condition of equation (6) applies only to farmers with unlimited capital. Farmers with limited capital and with alternative investment opportunities available for their capital may find it more profitable to invest only a portion of their limited capital in fertilizer. Hence, under the conditions of limited capital, the optimum rate of fertilizer use will generally be somewhat less than that specified by equation (6). The optimum rate of fertilizer use under conditions of limited capital will be discussed more in detail in a later section.

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\(^1\) The optimum rate of fertilizer use as specified by the condition of equation (6) applies only to farmers with unlimited capital. Farmers with limited capital and with alternative investment opportunities available for their capital may find it more profitable to invest only a portion of their limited capital in fertilizer. Hence, under the conditions of limited capital, the optimum rate of fertilizer use will generally be somewhat less than that specified by equation (6). The optimum rate of fertilizer use under conditions of limited capital will be discussed more in detail in a later section.
attaining the added crop yield as shown in equation (7a).

\[
\left(7\right) \quad \frac{\Delta C}{\Delta F_1} > \frac{F_i}{P} \quad \Delta C \cdot P > \Delta F_1 \cdot F_1 \quad C
\]

Hence it is profitable to increase the use of the fertilizer nutrient \( F_1 \) until the condition specified in equation (6) or equation (6a) is met. However, if the marginal physical product of the fertilizer nutrient \( F_1 \) is less than the fertilizer crop price ratio as shown in equation (8) below, then the value of the added crop yield will be less than the cost of attaining that yield as shown in equation (8a).

\[
\left(8\right) \quad \frac{\Delta C}{\Delta F_1} < \frac{F_i}{P} \quad \Delta C \cdot P < \Delta F_1 \cdot F_1 \quad C
\]

Since added costs now exceed added returns, profits can be increased by reducing the amount of the fertilizer nutrient \( F_1 \) used until the condition specified in equation (6) or (6a) is satisfied.

The optimum rate of fertilizer use for a single-variable fertilizer nutrient can be illustrated geometrically as in Figure 10. The curve OR, in Figure 10, is the same fertilizer crop response curve presented in Figure 2. As mentioned previously, the slope of this curve is the marginal physical product of the fertilizer nutrient \( F_1 \), or \( \frac{\Delta C}{\Delta F_1} \). The curve OM depicts the exchange value between the fertilizer nutrient \( F_1 \) and the crop. For example, if the price of the crop is $1.20 per bushel and the price of the fertilizer nutrient \( F_1 \) is 10 cents per pound, then the curve OM will pass through the origin and a point \( P \), where 1 bushel of the crop will purchase 12 pounds of the fertilizer nutrient \( F_1 \). Thus the slope of the curve OM represents the fertilizer crop price ratio \( \frac{F_1}{P} \cdot C \).

According to equation (6), the optimum rate of fertilizer use occurs where the marginal physical product of the fertilizer nutrient \( F_1 \) \( \left( \frac{\Delta C}{\Delta F_1} \right) \) is equal to the fertilizer crop price ratio \( \frac{F_1}{P} \cdot C \).

The optimum of fertilizer use can be illustrated graphically by constructing a curve O'M' that is parallel to the cost line OM and tangent to the fertilizer crop-response curve OR. At the point B, where the two curves are tangent, they also have equal slope, which satisfies the condition for the optimum rate of fertilizer use specified in equation (6). In the example in Figure 10, OF pounds of the fertilizer nutrient \( F_1 \) will be used to produce FB bushels of the crop. GF represents the number of bushels of the total crop FB that is required to pay for OF pounds of the fertilizer nutrient \( F_1 \). Thus GF represents in terms of crop yield the cost of OF pounds of the fertilizer nutrient \( F_1 \), while BG represents in terms of crop yield the returns above fertilizer costs that are obtained from using OF pounds of the fertilizer nutrient \( F_1 \).

If less than OF pounds of the fertilizer nutrient \( F_1 \) are used, then the marginal physical product of the nutrient
$F_1$ will be greater than the fertilizer crop price ratio as stated in equation (7). However, if more than $OF$ pounds of the nutrient $F_1$ are used, then the marginal physical product of the nutrient $F_1$ will be less than the fertilizer crop price ratio as stated in equation (8).

The geometric illustration in Figure 10 can also be used to show how a change in the fertilizer crop price ratio affects the optimum rate of fertilizer use. For example, if the price of fertilizer is 20 cents per pound instead of 10 cents per pound, then the line ON becomes the relevant cost line, and the point Q indicates that now 1 bushel of the crop will purchase only 6 pounds of the fertilizer. The crop yield is indicated at point C, where the line O"N" is tangent to the fertilizer crop response curve. The optimum quantity of fertilizer that should be used is OH pounds. Thus the lower the price of fertilizer relative to the price of the crop, the higher becomes the optimum rate of fertilizer use, while the higher the price of fertilizer relative to the price of the crop, the lower becomes the optimum rate of fertilizer use.

![Diagram](image-url)
In the case of a given fertilizer mix (a fixed combination of several nutrients), the fertilizer crop response curve, as mentioned previously, will also be of the form illustrated in Figure 2 and used in the example in Figure 10. Therefore the concepts that have been discussed for the single-variable nutrient will apply equally to an analysis of a given fertilizer mix.

**The Optimum Combination of Two-or-More-Variable Nutrients and the Optimum Level of Fertilizer Use**

When two or more fertilizer nutrients are variable, a farmer must: (1) decide what combination of fertilizer nutrients will minimize costs for any given level of crop yield, then (2) determine the level of fertilizer use which will be most profitable considering the least-cost combinations of the fertilizer nutrients.

The optimum combination of two fertilizer nutrients for any given level of crop yield is achieved when the condition specified in equation (9) below is attained. The ratio $\frac{\Delta F_2}{\Delta F_1}$, as mentioned previously, is the marginal rate of substitution of fertilizer nutrient $F_1$ for fertilizer nutrient $F_2$; $P$ is the price per unit of the nutrient $F_1$; and $P$ is the price per unit of the nutrient $F_2$.

\[
\frac{\Delta F_2}{\Delta F_1} = \left(\frac{P_{F_1}}{\Delta F_2} \cdot \frac{P_{F_2}}{\Delta F_1}\right) F_1
\]

According to equation (9) the optimum combination of two fertilizer nutrients for any given level of crop yield is attained when the marginal rate of substitution of fertilizer nutrient $F_1$ for nutrient $F_2$, $\left(\frac{\Delta F_2}{\Delta F_1}\right)$, is equal to the inverse price ratio of the two nutrients, $\left(\frac{P_{F_1}}{P_{F_2}}\right)$, which can be derived from equation (9), indicates that the optimum combination of two fertilizer nutrients occurs when the value of the replaced nutrient $F_2$ is equal to the value of the added nutrient $F_1$.

Any change in the fertilizer price ratio $\left(\frac{P_{F_1}}{P_{F_2}}\right)$ will alter the optimum combination of the fertilizer nutrients. For, if the marginal rate of substitution of nutrient $F_1$ for nutrient $F_2$ is greater than the price ratio, as shown in equation (10) below, then the value of the replaced nutrient $F_2$ will be greater than the value of the added nutrient $F_1$ as shown in equation (10a).

\[
\frac{\Delta F_2}{\Delta F_1} > \left(\frac{P_{F_1}}{P_{F_2}}\right) F_1
\]

Hence it is profitable, for any given level of crop yield, to substitute fertilizer nutrient $F_1$ for nutrient $F_2$ until
the condition specified in equation (9) is met. However, if the marginal rate of substitution of nutrient \( F_1 \) for the nutrient \( F_2 \) is less than the fertilizer price ratio as shown in equation (11) below, then the value of the replaced nutrient \( F_2 \) will be less than the value of the added nutrient \( F_1 \), as shown in equation (11a).

\[
\frac{\Delta F_2}{\Delta F_1} < \frac{F_1}{F_2} \quad \text{and} \quad \frac{\Delta F_2 P}{\Delta F_1 P} < \frac{F_1 P}{F_2 P} \quad \text{(11a)}
\]

\[
\frac{\Delta F_2}{\Delta F_1} < \frac{F_1}{F_2} \quad \text{(11)}
\]

Since the value of the replaced nutrient \( F_2 \) is now less than the value of the added nutrient \( F_1 \), profits can be increased by substituting the nutrient \( F_2 \) for the nutrient \( F_1 \), until the condition specified in equation (9) is satisfied.

The optimum combination of fertilizer nutrients for any given level of crop yield can be illustrated geometrically as shown in Figure 11. The crop-yield isouquant map displayed in Figure 11 is similar to the isouquant map shown in Figure 8. The isouquants are denoted by crop yields of 40, 60, 80, and 100 bushels per acre. As mentioned previously, the slope of any given isouquant is the marginal rate of substitution of nutrient \( F_1 \) for nutrient \( F_2 \). The curve CL depicts the fertilizer price ratio \( \frac{F_1}{F_2} \) and is referred to as an isocost (equal cost) line, since it denotes all the possible alternative combinations of the nutrients, \( F_1 \) and \( F_2 \), that could be purchased for the same expenditure. For example, if the price of the nutrient \( F_1 \) is 10c per pound and the price of the nutrient \( F_2 \) is 15c per pound, then the isocost line CL indicates that an expenditure of \$3.00 will purchase either (1) 20 lbs. of the nutrient \( F_2 \), or (2) 30 lbs. of the nutrient \( F_1 \), or (3) some combination of the fertilizer nutrients \( F_1 \) and \( F_2 \), as indicated by the line CL. The slope of the isocost line CL represents the price ratio \( \frac{F_1}{F_2} \). For example,

\[
\frac{F_1}{F_2} = \frac{\$0.10}{\$0.15} = \frac{2}{3},
\]

and the slope of the isocost line CL in Figure 11, 20 lbs. \( = \frac{2}{3} \). 30 lbs. \( = \frac{2}{3} \)

According to equation (9) the optimum combination of fertilizer nutrients for any given level of crop yield occurs where the marginal rate of substitution of the nutrient \( F_1 \) for the nutrient \( F_2 \) \( \frac{\Delta F_2}{\Delta F_1} \) is equal to the fertilizer price ratio \( \frac{F_1}{F_2} \). Since the slope of a crop-yield isouquant is the marginal rate of substitution of the nutrient \( F_1 \) for the nutrient \( F_2 \) and the slope of the isocost line is the inverse price ratio, then the point where the two curves are tangent indicates the optimum combination of the nutrient \( F_1 \) and \( F_2 \) for that given level of crop yield indicated by the crop-yield isouquant. Point M, in Figure 11, which is the point of tangency between the isocost line CL and the 40-bushel crop-yield isouquant, indicates the
least-cost combination of the nutrients $F_1$ and $F_2$ that will produce a crop yield of 40 bushels.

To determine what would be the least-cost combination of the fertilizer nutrients $F_1$ and $F_2$ for a crop yield of 100 bushels, an isocost line $CL'$ is constructed tangent to the 100-bushel crop-yield isoquant and parallel to the isocost line $CL$. Point $P$ indicates the least-cost combination of the nutrients $F_1$ and $F_2$ to yield a 100-bushel crop. Any other combination on the isocost line $CL'$ would produce less crop yield at the same cost. For example, the combination of the nutrient $F_1$ and $F_2$ indicated at point $S$ would produce only 80 bushels of crop yield at the same cost as the fertilizer combination at point $P$ that will produce 100 bushels. The least-cost combination of the nutrients $F_1$ and $F_2$ for different levels of crop yield could be found in a similar manner.
If the points on successive crop-yield isoquants, where the marginal rate of substitution of the fertilizer nutrient $F_1$ for the nutrient $F_2$ is equal to their inverse price ratio, are connected, they will form the line MP in Figure 11. The line MP is an *isocline* since it connects the points on successive crop-yield isoquants of equal slope or equal marginal rates of substitution. However, this line is a special kind of isocline, since it not only indicates the path of equal marginal rates of substitution, where the marginal rate of substitution is equal to a given fertilizer price ratio

$$\left( \frac{P}{F_1} = \frac{2}{3} \right).$$

Therefore the line MP is technically referred to as an *expansion path*, since it indicates for a given set of fertilizer prices the least-cost fertilizer combinations and the levels of fertilizer use for various levels of crop yield.

The optimum combination or the least-cost combination of the fertilizer nutrients $F_1$ and $F_2$ can now be determined for any given level of crop yield. However, the most profitable level of crop yield or the optimum level of fertilization still is not determined. In other words, the expansion path (MP in Figure 11) indicates the least-cost combination of the nutrient $F_1$ and $F_2$ for various levels of crop yield and as such it is the path of least cost along which crop yields should be expanded. However, the expansion path does not indicate which least-cost combination of nutrients along the path will be the most profitable.

In the case of the single-variable nutrient, discussed previously, the optimum level of fertilizer use was found to occur where the value of the added crop yield is equal to the cost of the incremental input (equation 6a). The most profitable level of fertilizer use for the two-variable nutrient can be defined in a similar manner except that there are now two variable inputs instead of only one. Thus the optimum level of fertilizer use or the most profitable level of crop yield in the two-variable case occurs where the value of the added crop yield is equal to the cost of the incremental inputs when they are combined in the least-cost manner specified earlier by the expansion path. The condition that specifies the optimum level of fertilization can be stated in an algebraic form as in equation (12).

$$\frac{\Delta C}{\Delta F_1} \frac{P}{C} = \frac{\Delta C}{\Delta F_2} \frac{P}{C} = 1.0$$

The numerators in equation (12) are the value of the marginal products of the fertilizer nutrients $F_1$ and $F_2$, respectively, while the denominators are their respective prices. According to the condition in equation (12), profits will be maximized when the cost of the incremental input of each fertilizer nutrient (i.e., the marginal cost) is equal to its marginal physical product, multiplied by the price of the crop ($P$).
ECONOMIC ANALYSIS OF NITROGEN USE ON IRRIGATED BARLEY

The basic relationships and principles for economic analysis of fertilizer use were presented earlier in this report in order that they might serve as a guide for empirical economic analysis of fertilizer use. The purpose of this section is not only to illustrate the use of some of the principles discussed in the preceding sections, but also to provide findings that can serve as the basis for making economic recommendations to farmers on the use of nitrogen fertilizer on barley.

Experimental Procedure

The data for this economic analysis are compiled from field experiments conducted during the 3-year period 1959, 1960, and 1961.\(^1\) The experiments were of similar design, namely, a randomized complete-block design with four replications. The experiments were conducted under irrigated conditions near Powell, Wyoming, in the Big Horn Basin.

In all the experiments, nitrogen was applied in the ammoniate form at the rates of 0, 20, 40, 60, 80, 100, 120, and 140 lbs. per acre. Each year the experiments were conducted at two separate farm sites in the same general area. Moreover, at least 100 lbs. of \(P_2O_5\) was applied to each experimental plot each year for assurance that this fertilizer nutrient would not be deficient at some farm sites and thus contribute to differences in barley yields.\(^2\)

Derivation of the Fertilizer Crop Response Function

Analysis of the experimental data, by means of analysis-of-variance methods, revealed for each experiment, at each farm site, that nitrogen fertilizer had a highly significant effect on barley yields. Moreover, the analysis-of-variance techniques revealed that for any given year there was a significant difference between the barley yields at the two farm sites. However, the interaction effect between farm sites and nitrogen fertilizer for any one year was non-significant (Appendix Tables 1, 2, and 3). The non-significant interaction effect suggests that there may be no difference between farm sites in the response of barley yields to nitrogen fertilizer.\(^3\) If there is no difference between farm sites, for any given year, in the response of barley yields to nitrogen fertilizer (i.e., no interaction effect between farm sites and nitrogen fertilizer), then the significant difference in crop yields between farm sites is due to

\(^1\) Data from field trials conducted by the Soils Section of the Plant Science Division, University of Wyoming, in cooperation with the Phillips Petroleum Company.

\(^2\) Hiland barley was the variety planted at all the farm sites for all three years except in 1960, when at one of the farm sites Frontier was planted.

\(^3\) To help determine whether the data from the two farm sites for each year could be pooled, "t" tests of the differences between corresponding regression coefficients were also made. The results were similar to those obtained from the analysis of variance.
differences in the absolute level of barley yields.

Since there was no evidence of a difference in the response of barley yields to nitrogen fertilizer between farm sites for any one year, regression equations were computed for the pooled data for each year. The nitrogen barley-response functions for each of the three years (1959, 1960, and 1961) are given in the following 3 equations, respectively, and plotted in Figure 12.\(^4\)

\[
\begin{align*}
(12) \quad B &= 50.0612 + 0.6261N - 0.0017N^2 \\
(13) \quad B &= 52.3134 + 0.3726N - 0.0012N^2 \\
(14) \quad B &= 69.4791 + 0.4064N - 0.0016N^2 
\end{align*}
\]

B refers to the estimated yield of barley in bushels per acre and N depicts the level of nitrogen application in pounds per acre.

Farm sites could have been included in the regression equations for each year in order to increase the precision of predicting the actual yields for each farm site. However, since predicting the actual yields at each farm site is secondary to predicting the response of barley yields to nitrogen-fertilizer inputs, farm sites have not been included in the regression equations. In the economic analysis which follows, the primary interest is in the slopes of the production curves rather than in the absolute level of barley yields.

### Optimum Rates of Nitrogen Fertilization

As discussed previously, the economic optimum level of nitrogen fertilization is estimated by equating the marginal product of nitrogen \(\frac{\partial B}{\partial N}\) with the inverse price ratio \(\frac{P_N}{P_B}\), where \(P_N\) refers to the price of actual nitrogen in cents per pound and \(P_B\) refers to the price of barley in cents per bushel. Optimum rates of nitrogen fertilization have been computed for the average prices of barley and nitrogen for each of the 3 years used in this study. The cost of nitrogen used in estimating these optimum rates was $0.13 per pound of actual nitrogen, while barley was priced at $0.90 per bushel, which is the average price of barley for the 5-year period 1957-1961.\(^5\)

\(^4\) The probability levels of the t values for the equations are given in Appendix Table 4.

# TABLE 1. Estimated Increase in Returns per Acre After the Cost of Nitrogen from Using the Optimum Level of Fertilizer each Year. a/ 

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>1959</th>
<th>1960</th>
<th>1961</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen (optimum)</td>
<td>Pounds</td>
<td>142</td>
<td>95</td>
<td>82</td>
</tr>
<tr>
<td>Crop yield with nitrogen</td>
<td>Bu.</td>
<td>105</td>
<td>77</td>
<td>92</td>
</tr>
<tr>
<td>Crop yield without nitrogen</td>
<td>Bu.</td>
<td>50</td>
<td>52</td>
<td>69</td>
</tr>
<tr>
<td>Increase in yield</td>
<td>Bu.</td>
<td>55</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>Value of increased yield</td>
<td>Dollars</td>
<td>$49.50</td>
<td>$22.50</td>
<td>$20.70</td>
</tr>
<tr>
<td>Cost of nitrogen</td>
<td>Dollars</td>
<td>$18.46</td>
<td>$12.35</td>
<td>$10.66</td>
</tr>
<tr>
<td>Increase in the returns/acre after the cost of nitrogen</td>
<td>Dollars</td>
<td>$31.04</td>
<td>$10.15</td>
<td>$10.04</td>
</tr>
</tbody>
</table>

a/ Nitrogen is valued at $0.13 per pound of actual nitrogen derived from ammonium nitrate; barley is valued at $0.90 per bushel.

If nitrogen had been applied each year at the optimum rate specified for each year, then the increased returns per acre from nitrogen fertilization would have averaged $17.08 per acre for the 3-year period above the cost of the nitrogen fertilizer. The increased returns per acre are the returns per acre over the returns without nitrogen fertilization.

The data, as presented in Table 1, indicate that the economic optimum level of nitrogen fertilization tends to vary from year to year, even though prices may remain constant. The uncertainty of the physical-response curve for any one year is also illustrated in Figure 12, where the response curves have shifted in each year of the 3-year period.

Hence one major uncertainty in making fertilizer recommendations is the uncertainty as to the exact nature of the physical-response function.

Therefore, since it is not possible at the beginning of the production season to predict accurately what will be the exact nature of the physical-response function for that season, fertilizer recommendations are generally determined from past yield experiences. The method that is generally used is to pool the different experiments and derive an "average" physical-response function. Fertilizer recommendations are then derived using this average physical-response function.6/

The average nitrogen/barley-response function for the 3-year period 1959-1961 is given in the following equation and also plotted in Figure 12.7/

\[(15) \quad B = 57.2841 + 0.4684N - 0.0015 N^2 \]

B refers to the estimated yield of barley in bushels per acre and N depicts the level of nitrogen application in pounds per acre.

6/ Fertilizer recommendations based on the "average" physical-response function will result in too little fertilizer being applied for profit maximization in a better-than-average crop year and too much in a poorer-than-average crop year.

7/ The probability levels of the t values for the equation are given in Appendix Table 4.
Optimum Rates of Fertilization Under Average Price and Yield Conditions

The optimum rate of nitrogen fertilization, based on the average physical-response function and on the average prices for barley and nitrogen, is 108 lbs. of nitrogen per acre. Applying nitrogen fertilizer at this average optimum rate would return an average annual profit of $15.74. However, had this average rate of 108 lbs. been applied each year instead of the optimum rate for each specific year, the average annual profits would have been reduced $1.34 per acre.

If a rate of nitrogen application is used which is more or less than the recommended optimum rate, profits will be reduced. However, for only small deviations from the optimum rate, profits will not be reduced an appreciable amount, as shown in Table 2.

TABLE 2. Increase in Returns per Acre after the Cost of Nitrogen for Various Levels of Nitrogen Use. a/

<table>
<thead>
<tr>
<th>Pounds of nitrogen per acre</th>
<th>Dollars</th>
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<tr>
<td>20</td>
<td>$ 5.29</td>
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<tr>
<td>30</td>
<td>7.53</td>
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<td>40</td>
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<td>80</td>
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<tr>
<td>90</td>
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<tr>
<td>100</td>
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<tr>
<td>110</td>
<td>15.74b/</td>
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<tr>
<td>120</td>
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<td>130</td>
<td>15.09</td>
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<tr>
<td>140</td>
<td>14.33</td>
</tr>
</tbody>
</table>

a/ Nitrogen is valued at $0.13 per pound of actual nitrogen derived from ammonium nitrate. Barley is valued at $0.90 per bushel.
b/ The exact optimum level of nitrogen application for the average nitrogen barley response function was 108 pounds of nitrogen.
Hence a farmer has some margin of choice in the rate of fertilization he should use. If he is short of operating capital or anticipates a less-than-average crop year, he may apply the recommended optimum rate or even apply slightly less nitrogen than the optimum rate. However, if operating capital is available and he anticipates a better than average crop year, he may actual-

ly exceed the recommended rate of fertilization in hopes of profiting from a better-than-average crop year. Thus the decision as to the rate of nitrogen to use depends upon the individual farmer and his ability to bear risk. His risk-bearing ability will depend upon his aversion to risk-taking and his capital and equity position.

Optimum Rates of Fertilization
Under Alternative Price Ratios

The economic optimum rate of fertilization depends upon not only the fertilizer-response function but also on the fertilizer barley price ratio. However, since the prices of barley and nitrogen are subject to change, the ratio of their price may also change, which in turn may alter the economic optimum rate of fertilization. These rates of fertilization for various alternative price conditions are shown in Table 3. The data indicate that, as the price of fertilizer increases for any given price of barley, the economic optimum rate of nitrogen fertilization decreases. For example, if the price of barley is $ .90 per bushel and the price of nitrogen fertilizer is $.10 per pound, then the optimum rate of fertilization is 120 lbs. of nitrogen per acre. If the price of nitrogen moves to $.15 per pound, then the optimum rate of fertilization is 101 lbs. However, when the price of barley increases for any given price of nitro-

<table>
<thead>
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<th>Price of nitrogen per lb.</th>
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<th>.90</th>
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<td>10 lbs. of nitrogen per acre</td>
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gen, then the economic optimum rate of nitrogen fertilization increases.

In making economic-optimum fertilizer recommendations for various nitrogen and barley prices, it is the ratio of their prices that is relevant and not the absolute prices. For example in Table 3, when the price of nitrogen is $.06 per pound and the price of barley is $.60 per bushel (the ratio of the two prices is \( \frac{.06}{.60} = .1 \)), the optimum rate of fertilization is 123 lbs. of nitrogen. Similarly, when the price of nitrogen is $.10 and the price of barley is $1.00 (the ratio of the two prices is \( \frac{.10}{1.00} = .1 \)), the optimum rate of fertilization is again 123 lbs. Therefore, since it is this ratio of the nitrogen price to the barley price and not the absolute prices that is needed to determine the optimum fertilization rates, then it is possible to construct a price-ratio map as shown in Figure 13, in order to simplify fertilizer recommendations under various alternative nitrogen and barley-price conditions.

Given the price of nitrogen and the price of barley, the optimum rate of fertilization can be determined in the following manner: Locate the price of nitrogen on the horizontal scale and the expected price of barley on the vertical scale. The point on the graph where perpendicular lines from the two prices intersect will denote the rate of fertilizer to apply.

For example, assume the price of nitrogen is $.17 per pound and the expected price of barley is $1.10 per bushel, then the optimum rate would be between 100 and 110 lbs. per acre. If greater precision than the 10-lb. interval is desired, then it is necessary to estimate the distance between the two rays where the intersection point of the two price lines occurs. In the above example the point occurs halfway between the 100 and 110-lb. rays. Therefore the optimum rate is approximately 105 lbs. per acre. Generally such precision will not be required for most fertilizer recommendations.

### Alternative Investment Opportunities

Nitrogen fertilizer recommendations as discussed in the preceding section consider fertilization of barley as a production practice and an opportunity for investment that is completely isolated from the rest of the farm business. Fertilizer use is, however, only one of many practices and investment opportunities that are available to the farmer. Therefore the farmer should consider fertilizer use as an integrated part of his entire farming operations. The optimum rates of nitrogen fertilization as presented in Table 3 and Figure 13 assume that the farmer has sufficient capital available to carry production to the point where added costs are just equal to added returns. At this point of production the last dollar invested in fertilizer will increase barley returns by one dollar. Therefore the rate of return on the last dollar invested in fer-
FIG. 13—Price-ratio map for determining the optimum pounds of nitrogen to apply per acre.
tilizer (the marginal dollar) is zero.

If a farmer is limited on capital and has alternative investment opportunities with a prospect of a higher rate of return on the investment, then, of course, he should pursue these opportunities first before he fertilizes barley at the “optimum” rates as presented in Table 3 and Figure 13. For example, assume a farmer has an opportunity to earn a 30 percent return from buying additional protein feed for a feeding enterprise. The farmer should then invest in protein feed before he fertilizes barley at the “optimum” rates discussed above. For a farmer with limited funds to maximize the returns to his farm, he must equate the marginal productivity of capital (i.e., the rate of return on the last dollar invested) in all available investment opportunities. Thus fertilizer recommendations that consider the marginal productivity of capital invested in fertilizer will allow for the rate of return on the last dollar (the marginal dollar) invested in fertilizer to be equated with the rate of return in all other investments.

The price-ratio map in Figure 14 has been prepared to facilitate the estimation of the level of nitrogen fertilizer use that will provide a specified rate of return on the last dollar invested in fertilizer. Given the level of nitrogen fertilization that would be optimum under conditions of unlimited capital as discussed in the previous section, the level of fertilization that will return a specified rate of return on the marginal dollar invested in fertilizer may be determined in the following manner: Determine the “optimum” level of nitrogen fertilization in Figure 13 for the particular price conditions under consideration. (The rate of return on the marginal dollar invested for this optimum rate is zero.) After determining the “optimum” level, refer to Figure 14. Locate on the horizontal scale the “optimum” level as determined from Figure 13. Then follow the diagonal line that intersects the horizontal scale at the point of the “optimum” level until the desired rate of return on the last dollar invested in fertilizer, as indicated on the vertical scale, has been achieved. Then read directly below, on the horizontal scale, the level of fertilization that will return this desired rate of return for the particular price conditions under consideration.

As an example, consider the example used in the previous section, where nitrogen was priced at $.17 per pound and barley at $1.10 per bushel. The optimum level of fertilization was found to be 105 lbs. per acre. Assume a farmer has alternative investment opportunities that will provide him with a 40 percent marginal return. To get a 40 percent return of his investment in fertilizer he would only apply approximately 85 lbs. per acre instead of the 105 lbs. To obtain a 100 percent return on his investment he would cut his nitrogen application to approximately 52 lbs. per acre.

Fertilizer recommendations need to be adjusted to each individual farmer’s needs. There is no such “standard” recommendation that can be made to cover all cases. Recommendations need to be made so that farmers can make decisions as to fertilizer use that conform with the cost/price relationships, their particular capital situation, and their uncertainty and risk-bearing ability.
FIG. 14—Price-ratio map for determining the level of nitrogen fertilization for a designed percentage return on the last dollar invested in nitrogen fertilizer for various price ratios.
SUMMARY AND CONCLUSIONS

This report is divided into two parts. The emphasis in the first part is placed on the basic agronomic and economic relationships and principles of fertilizer use in the hope that these may serve as a guide for future fertilizer crop research. The second part is concerned with economic analysis of nitrogen use on irrigated barley. Experiments were conducted in the Big Horn Basin near Powell, Wyoming, for a 3-year period, 1959 through 1961. The experimental design was a randomized complete block with four replications.

Data from the nitrogen fertilizer experiments were used to estimate nitrogen barley response functions for each of the 3 years and an average function for all 3 years. The average nitrogen barley function was used for the economic analysis made in this study.

Under average price conditions, when barley is worth $.90 per bushel and nitrogen cost $.13 per pound for actual nitrogen, the “optimum” level of nitrogen fertilization is 108 lbs. per acre. However, the optimum level of fertilization will generally change under different nitrogen barley price ratios. A graph has been prepared which provides a simple and convenient method of determining the “optimum” level of nitrogen fertilization under different nitrogen barley price conditions.

Since fertilizer use is only one of many practices and investment opportunities that are available to the farmer, the farmer should consider fertilizer use as an integrated part of his entire farming operations. Thus, to maximize returns to the farm the marginal productivity of capital should be equal in all available investment opportunities. A graph has been prepared to facilitate the estimate of the level of nitrogen fertilization that will return a specified rate of return to capital invested in nitrogen fertilizer.

APPENDIX TABLE 1. Analysis of Variance of Irrigated Barley Yields
Based on Experimental Results at Powell, Wyoming, for 1959

<table>
<thead>
<tr>
<th>SOURCE OF VARIATION</th>
<th>Degrees of freedom</th>
<th>Sum of square</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replications</td>
<td>3</td>
<td>1,459.797</td>
<td>486.60</td>
<td>2.79</td>
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<tr>
<td>Farm sites</td>
<td>1</td>
<td>813.676</td>
<td>813.68</td>
<td>4.66*</td>
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<tr>
<td>Nitrogen</td>
<td>7</td>
<td>21,672.362</td>
<td>3,096.05</td>
<td>17.75**</td>
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<tr>
<td>Sites x nitrogen</td>
<td>7</td>
<td>1,993.293</td>
<td>284.76</td>
<td>1.63</td>
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<tr>
<td>Experimental error</td>
<td>45</td>
<td>7,847.083</td>
<td>174.38</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>33,786.211</td>
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<td></td>
</tr>
</tbody>
</table>

* Probability level ≤ 0.05.
** Probability level ≤ 0.01.
### APPENDIX TABLE 2. Analysis of Variance of Irrigated Barley Yields Based on Experimental Results at Powell, Wyoming, for 1960

<table>
<thead>
<tr>
<th>SOURCE OF VARIATION</th>
<th>Degrees of freedom</th>
<th>Sum of square</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replications</td>
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<td>103,503</td>
<td>34.50</td>
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<tr>
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<td>763.76</td>
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<tr>
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<td>6,356,859</td>
<td>908.12</td>
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<tr>
<td>Sites x nitrogen</td>
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<td>550,710</td>
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<td>1.614</td>
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<tr>
<td>Experimental error</td>
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<td><strong>Total</strong></td>
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<td>9,968.042</td>
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</tbody>
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* Probability level ≤ 0.05.
** Probability level ≤ 0.01.

### APPENDIX TABLE 3. Analysis of Variance of Irrigated Barley Yields Based on Experimental Results at Powell, Wyoming, for 1961

<table>
<thead>
<tr>
<th>SOURCE OF VARIATION</th>
<th>Degrees of freedom</th>
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<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replications</td>
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<td>438,593</td>
<td>146.20</td>
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<tr>
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<td>445,248</td>
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<tr>
<td>Experimental error</td>
<td>45</td>
<td>2,148,591</td>
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<td><strong>Total</strong></td>
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* Probability level ≤ 0.05
** Probability level ≤ 0.01

### APPENDIX TABLE 4. Regression Values and Statistical Tests for Quadratic Functions Fitted to Experimental Results at Powell, Wyoming

<table>
<thead>
<tr>
<th>EQUATION</th>
<th>$b_1$</th>
<th>PROBABILITY LEVEL FOR $b_1$</th>
<th>$b_2$</th>
<th>PROBABILITY LEVEL FOR $b_2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>.6261</td>
<td>$P &lt; 0.001$</td>
<td>-.0017</td>
<td>$0.05 &lt; P &lt; 0.10$</td>
<td>.6019</td>
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<tr>
<td>13</td>
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<td>$P &lt; 0.001$</td>
<td>-.0012</td>
<td>$0.10 &lt; P &lt; 0.20$</td>
<td>.6271</td>
</tr>
<tr>
<td>14</td>
<td>.4064</td>
<td>$0.01 &lt; P &lt; 0.05$</td>
<td>-.0016</td>
<td>$0.10 &lt; P &lt; 0.20$</td>
<td>.2156</td>
</tr>
<tr>
<td>15</td>
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<td>$P &lt; 0.001$</td>
<td>-.0015</td>
<td>$0.01 &lt; P &lt; 0.05$</td>
<td>.3739</td>
</tr>
</tbody>
</table>
SELECTED LITERATURE


2. Berry, Russell L. *Most Profitable Use of Fertilizer on Corn, Oats, and Wheat in South Dakota*, Agricultural Economics Pamphlet 69, South Dakota (College Station) Agric. Experiment Station, 1956.


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