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Stephen W. Drury
Department of Mathematics & Statistics, McGill University, Montreal, drury@math.mcgill.ca

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A COUNTEREXAMPLE TO A QUESTION OF BAPAT AND SUNDER

STEPHEN W. DRURY

Abstract. A counterexample to a question of Bapat and Sunder is presented.

Key words. Permanent, Hadamard product, Oppenheim’s inequality.

AMS subject classifications. 15A15.

1. Introduction. In [1], Bapat and Sunder raise the question of whether the inequality

$$\text{per}(A \odot B) \leq \text{per}(A) \prod_{j=1}^{n} b_{jj} \quad (1.1)$$

holds for positive semidefinite $n \times n$ matrices $A$ and $B$. The quantity $\text{per}(A)$ denotes the permanent of $A$ and the notation $A \odot B$ is for the Hadamard (entrywise) product of $A$ and $B$. This is the permanent version of Oppenheim’s inequality. It is the objective of this article to provide a counterexample. The question is related to two other questions:

- The permanent on top conjecture, recently disproved by Shchesnovich [4] which would have implied (1.1) had it been true.
- The inequality $\text{per}(A \odot B) \leq \text{per}(A)\text{per}(B)$ introduced by Chollet [2] and established in the case $n = 3$ by Gregorac and Hentzel [3]. This inequality would be a consequence of (1.1) had it been true. Chollet’s conjecture remains open. For a relatively recent discussion of Chollet’s conjecture, the reader may consult Zhang [5].

2. The counterexample. With $n = 7$, we take

$$A = \begin{pmatrix}
1 & 0 & 0 & e^{\frac{i\pi}{5}} & e^{\frac{2i\pi}{5}} & e^{\frac{3i\pi}{5}} & e^{\frac{4i\pi}{5} \\
0 & 1 & e^{-\frac{i\pi}{5}} & e^{-\frac{2i\pi}{5}} & e^{-\frac{3i\pi}{5}} & e^{-\frac{4i\pi}{5}} & e^{-\frac{5i\pi}{5}} \\
ee^{\frac{i\pi}{5}} & e^{-\frac{i\pi}{5}} & 1 & e^{\frac{2i\pi}{5}} & e^{\frac{3i\pi}{5}} & e^{\frac{4i\pi}{5}} & e^{\frac{5i\pi}{5}} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}$$


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\†Department of Mathematics and Statistics, McGill University, Montreal, Canada H3A 0B9 (drury@math.mcgill.ca).
and $B = A^T$. Then it is easy to check that $A$ and $B$ are hermitian positive semidefinite matrices of rank two (with eigenvalue $\frac{7}{2}$ of multiplicity two) and that

$$\text{per}(A \circ B) = \frac{6185}{128},$$

that $\prod_{j=1}^{7} b_{jj} = 1$ and that $\text{per}(A) = 45$. We find that

$$\frac{\text{per}(A \circ B)}{\text{per}(A) \prod_{j=1}^{7} b_{jj}} = \frac{1237}{1152} > 1.$$