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Chaoqian Li
Yunnan University, lichaoqian@ynu.edu.cn

Mengting Gan

Shaorong Yang

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A NEW ERROR BOUND FOR LINEAR COMPLEMENTARITY PROBLEMS FOR B-MATRICES*

CHAOQIAN LI†, MENGTING GAN†, AND SHAORONG YANG†

Abstract. A new error bound for the linear complementarity problem is given when the involved matrix is a B-matrix. It is shown that this bound improves the corresponding result in [M. García-Esnaola and J.M. Peña. Error bounds for linear complementarity problems for B-matrices. *Appl. Math. Lett.*, 22:1071–1075, 2009.] in some cases, and that it is sharper than that in [C.Q. Li and Y.T. Li. Note on error bounds for linear complementarity problems for B-matrices. *Appl. Math. Lett.*, 57:108–113, 2016.].

Key words. Error bound, Linear complementarity problem, B-Matrix.

AMS subject classifications. 90C33, 65G50, 65F35.

1. Introduction. A linear complementarity problem LCP(M, q) tries to find a vector $x \in R^n$ such that

$$x \geq 0, \ \ Mx + q \geq 0, \ \ (Mx + q)^T x = 0, \tag{1.1}$$

where $M = [m_{ij}] \in R^{n\times n}$ and $q \in R^n$. The $LCP(M, q)$ has various applications in the Nash equilibrium point of a bimatrix game, the contact problem and the free boundary problem for journal bearing; for details, see [4, 5, 17].

It is well-known that the LCP(M, q) has a unique solution for any $q \in R^n$ if and only if M is a P-matrix [5]. Here, a matrix $M \in R^{n\times n}$ is called a P-matrix if all its principal minors are positive [6]. In [3], Chen and Xiang gave the following error bound of the LCP(M, q) when M is a P-matrix:

$$||x - x^*||_\infty \leq \max_{d\in[0,1]^n} ||(I - D + DM)^{-1}||_\infty ||r(x)||_\infty,$$

where x^* is the solution of the LCP(M, q), $r(x) = \min\{x, Mx + q\}$, $D = diag(d_i)$ with $0 \leq d_i \leq 1$, and the min operator $r(x)$ denotes the componentwise minimum of two vectors. If M satisfies certain structure, then some bounds of $\max_{d\in[0,1]^n} ||(I - D + DM)^{-1}||_\infty$ can be derived; for details, see [2, 7, 8, 10, 14] and references therein.

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†School of Mathematics and Statistics, Yunnan University, Kunming, Yunnan, 650091, P.R. China (lichaoqian@ynu.edu.cn).

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When M is a B-matrix introduced by Peña in [6] as a subclass of P-matrices, García-Esnaola and Peña in [10] presented the following upper bound which is only related with the entries of M. Here a real matrix $M = [m_{ij}] \in R^{n\times n}$ is called a B-matrix [6] if for each $i \in N = \{1, 2, \dots, n\}$,

$$\sum_{k\in N} m_{ik} > 0, \text{ and } \frac{1}{n}\left(\sum_{k\in N} m_{ik}\right) > m_{ij} \text{ for any } j \in N \text{ and } j \neq i. \tag{1.2}$$

THEOREM 1.1. [10, Theorem 2.2] *Let $M = [m_{ij}] \in R^{n\times n}$ be a B-matrix with the form*

$$M = B^+ + C, \tag{1.3}$$

where

$$B^+ = [b_{ij}] = \begin{bmatrix} m_{11} - r_1^+ & \cdots & m_{1n} - r_1^+ \\ \vdots & & \vdots \\ m_{n1} - r_n^+ & \cdots & m_{nn} - r_n^+ \end{bmatrix}, \tag{1.4}$$

and $r_i^+ = \max\{0, m_{ij} | j \neq i\}$. Then

$$\max_{d\in[0,1]^n} ||(I - D + DM)^{-1}||_\infty \leq \frac{n-1}{\min\{\beta, 1\}}, \tag{1.5}$$

where $\beta = \min_{i\in N}\{\beta_i\}$ and $\beta_i = b_{ii} - \sum_{j\neq i} |b_{ij}|$.

As shown in [15], if the diagonal dominance of B^+ is weak, i.e.,

$$\beta = \min_{i\in N}\{\beta_i\} = \min_{i\in N}\left\{ b_{ii} - \sum_{j\neq i} |b_{ij}| \right\}$$

is small, then the bound (1.5) may be very large when M is a B-matrix, which leads to that the estimate in (1.5) is always inaccurate, for details, see [15, 16]. To improve the bound (1.5), Li and Li [15] gave the following bound for $\max_{d\in[0,1]^n} ||(I - D + DM)^{-1}||_\infty$ when M is a B-matrix.

THEOREM 1.2. [15, Theorem 4] *Let $M = [m_{ij}] \in R^{n\times n}$ be a B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is the matrix of (1.4). Then*

$$\max_{d\in[0,1]^n} ||(I - D + DM)^{-1}||_\infty \leq \sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1}\left(1 + \frac{1}{\bar{\beta}_j}\sum_{k=j+1}^{n} |b_{jk}|\right), \tag{1.6}$$

Electronic Journal of Linear Algebra ISSN 1081-3810
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where $\bar{\beta}_i = b_{ii} - \sum\limits_{j=i+1}^{n} |b_{ij}| l_i(B^+)$, $l_k(B^+) = \max\limits_{k \le i \le n} \left\{ \frac{1}{|b_{ii}|} \sum\limits_{\substack{j=k, \\ j \ne i}}^{n} |b_{ij}| \right\}$ *and*

$$\prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}| \right) = 1 \;\; \text{if } i = 1.$$

Very recently, when M is a weakly chained diagonally dominant B-matrix, Li and Li [16] gave a bound for $\max\limits_{d \in [0,1]^n} ||(I - D + DM)^{-1}||_\infty$. This bound holds true for the case that M is a B-matrix because a B-matrix is a weakly chained diagonally dominant B-matrix [16].

THEOREM 1.3. [16, Corollary 1] *Let* $M = [m_{ij}] \in R^{n \times n}$ *be a* B*-matrix with the form* $M = B^+ + C$, *where* $B^+ = [b_{ij}]$ *is the matrix of (1.4). Then*

$$\max_{d \in [0,1]^n} ||(I - D + DM)^{-1}||_\infty \le \sum_{i=1}^{n} \left(\frac{n-1}{\min\{\tilde{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j} \right),$$

where $\tilde{\beta}_i = b_{ii} - \sum\limits_{j=i+1}^{n} |b_{ij}| > 0$ *and* $\prod\limits_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j} = 1$ *if* $i = 1$.

In this paper, we also focus on the error bound for the LCP(M, q), and gave a new bound for $\max\limits_{d \in [0,1]^n} ||(I - D + DM)^{-1}||_\infty$ when M is a B-matrix. It is shown that this bound is more effective to estimate $\max\limits_{d \in [0,1]^n} ||(I - D + DM)^{-1}||_\infty$ than that in Theorem 1.1, and sharper than those in Theorems 1.2 and 1.3.

2. Main results. We first recall some definitions. A matrix $A = [a_{ij}] \in C^{n \times n}$ is called a strictly diagonally dominant (SDD) matrix if for each $i \in N$, $|a_{ii}| > \sum\limits_{\substack{j=1, \\ j \ne i}}^{n} |a_{ij}|$. It is well-known that an SDD matrix is nonsingular [1]. A matrix $A = [a_{ij}]$ is called a Z-matrix if $a_{ij} \le 0$ for any $i \ne j$, and a nonsingular M-matrix if A is a Z-matrix with A^{-1} being nonnegative [1]. Next, several lemmas which will be used later are given.

LEMMA 2.1. [18, Theorem 3.2] *Let* $A = [a_{ij}] \in R^{n \times n}$ *be an* SDD M*-matrix. Then*

$$||A^{-1}||_\infty \le \sum_{i=1}^{n} \left(\frac{1}{a_{ii}(1 - u_i(A) l_i(A))} \prod_{j=1}^{i-1} \frac{1}{1 - u_j(A) l_j(A)} \right),$$

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where $u_i(A) = \frac{1}{|a_{ii}|} \sum\limits_{j=i+1}^{n} |a_{ij}|$, $l_k(A) = \max\limits_{k\le i\le n} \left\{ \frac{1}{|a_{ii}|} \sum\limits_{\substack{j=k,\\ j\neq i}}^{n} |a_{ij}| \right\}$ *and*

$$\prod_{j=1}^{i-1} \frac{1}{1-u_j(A)l_j(A)} = 1 \ \ if\ i=1.$$

Lemma 2.2. [15, Lemma 3] *Let $\gamma > 0$ and $\eta \geq 0$. Then for any $x \in [0,1]$,*

$$\frac{1}{1-x+\gamma x} \leq \frac{1}{\min\{\gamma,1\}} \tag{2.1}$$

and

$$\frac{\eta x}{1-x+\gamma x} \leq \frac{\eta}{\gamma}. \tag{2.2}$$

Lemma 2.3. [16, Lemma 5] *Let $A = [a_{ij}] \in R^{n\times n}$ with*

$$a_{ii} > \sum_{j=i+1}^{n} |a_{ij}| \ \ for\ each\ i \in N.$$

Then for any $x_i \in [0,1]$, $i \in N$,

$$\frac{1-x_i+a_{ii}x_i}{1-x_i+a_{ii}x_i - \sum\limits_{j=i+1}^{n} |a_{ij}|x_i} \leq \frac{a_{ii}}{a_{ii} - \sum\limits_{j=i+1}^{n} |a_{ij}|}.$$

Theorem 2.4. *Let $M = [m_{ij}] \in R^{n\times n}$ be a B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is the matrix of (1.4). Then*

$$\max_{d\in[0,1]^n} ||(I-D+DM)^{-1}||_\infty \leq \sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\bar{\beta}_j}, \tag{2.3}$$

where $\bar{\beta}_i$ is defined in Theorem 1.2 and $\prod\limits_{j=1}^{i-1} \frac{b_{jj}}{\bar{\beta}_j} = 1$ if $i = 1$.

Proof. Let $M_D = I - D + DM$. Then

$$M_D = I - D + DM = I - D + D(B^+ + C) = B_D^+ + C_D,$$

where $B_D^+ = I - D + DB^+ = [b_{ij}]$. Similarly to the proof of Theorem 2.2 in [10], we can obtain that B_D^+ is an SDD M-matrix with positive diagonal elements and $C_D = DC$, and that

$$||M_D^{-1}||_\infty \leq ||(I + (B_D^+)^{-1}C_D)^{-1}||_\infty ||(B_D^+)^{-1}||_\infty \leq (n-1)||(B_D^+)^{-1}||_\infty. \tag{2.4}$$

Electronic Journal of Linear Algebra ISSN 1081-3810
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Volume 31, pp. 476-484, July 2016

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By Lemma 2.1,

$$\|(B_D^+)^{-1}\|_\infty \leq \sum_{i=1}^{n}\left(\frac{1}{(1-d_i+b_{ii}d_i)\left(1-u_i(B_D^+)l_i(B_D^+)\right)} \times \prod_{j=1}^{i-1}\frac{1}{1-u_j(B_D^+)l_j(B_D^+)}\right), \tag{2.5}$$

where

$$u_i(B_D^+) = \frac{\sum\limits_{j=i+1}^{n}|b_{ij}|d_i}{1-d_i+b_{ii}d_i}, \text{ and } l_k(B_D^+) = \max_{k\leq i\leq n}\left\{\frac{\sum\limits_{\substack{j=k,\\ j\neq i}}^{n}|b_{ij}|d_i}{1-d_i+b_{ii}d_i}\right\}.$$

By Lemma 2.2, we can easily get that for each $k\in N$,

$$l_k(B_D^+) \leq \max_{k\leq i\leq n}\left\{\frac{1}{b_{ii}}\sum_{\substack{j=k,\\ j\neq i}}^{n}|b_{ij}|\right\} = l_k(B^+) < 1, \tag{2.6}$$

and that for each $i\in N$,

$$\begin{aligned}\frac{1}{(1-d_i+b_{ii}d_i)\left(1-u_i(B_D^+)l_i(B_D^+)\right)} &= \frac{1}{1-d_i+b_{ii}d_i-\sum\limits_{j=i+1}^{n}|b_{ij}|d_i l_i(B_D^+)}\\ &\leq \frac{1}{\min\left\{b_{ii}-\sum\limits_{j=i+1}^{n}|b_{ij}|l_i(B^+),1\right\}}\\ &= \frac{1}{\min\left\{\bar{\beta}_i,1\right\}}.\end{aligned} \tag{2.7}$$

Furthermore, by Lemma 2.3,

$$\frac{1}{1-u_i(B_D^+)l_i(B_D^+)} = \frac{1-d_i+b_{ii}d_i}{1-d_i+b_{ii}d_i-\sum\limits_{j=i+1}^{n}|b_{ij}|d_i l_i(B_D^+)} \leq \frac{b_{ii}}{\bar{\beta}_i}. \tag{2.8}$$

By (2.5), (2.6), (2.7) and (2.8), we have

$$\|(B_D^+)^{-1}\|_\infty \leq \frac{1}{\min\left\{\bar{\beta}_1,1\right\}} + \sum_{i=2}^{n}\frac{1}{\min\left\{\bar{\beta}_i,1\right\}}\prod_{j=1}^{i-1}\frac{b_{jj}}{\bar{\beta}_j}. \tag{2.9}$$

The conclusion follows from (2.4) and (2.9). $\square$

Electronic Journal of Linear Algebra ISSN 1081-3810
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Volume 31, pp. 476-484, July 2016

The comparisons of the bounds in Theorems 1.2, 1.3 and 2.4 are established as follows.

THEOREM 2.5. *Let $M = [m_{ij}] \in R^{n\times n}$ be a B-matrix with the form $M = B^+ + C$, where $B^+ = [b_{ij}]$ is the matrix of (1.4). Let $\bar{\beta}_i$ and $\tilde{\beta}_i$ be defined in Theorems 1.2 and 1.3 respectively. Then*

$$\sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\bar{\beta}_j} \leq \sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}|\right)$$
$$\leq \sum_{i=1}^{n} \left(\frac{n-1}{\min\{\tilde{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j}\right).$$

Proof. Note that

$$\tilde{\beta}_i = b_{ii} - \sum_{j=i+1}^{n} |b_{ij}|, \quad \bar{\beta}_i = b_{ii} - \sum_{j=i+1}^{n} |b_{ij}| l_i(B^+)$$

and $l_k(B^+) = \max\limits_{k\leq i\leq n} \left\{ \frac{1}{|b_{ii}|} \sum\limits_{\substack{j=k,\\ j\neq i}}^{n} |b_{ij}| \right\} < 1$. Hence, for each $i \in N$, $\tilde{\beta}_i \leq \bar{\beta}_i$ and

$$\frac{1}{\min\{\tilde{\beta}_i, 1\}} \geq \frac{1}{\min\{\bar{\beta}_i, 1\}}. \tag{2.10}$$

Meantime, for $j = 1, 2, \ldots, n-1$,

$$1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}| \leq 1 + \frac{1}{\tilde{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}| = \frac{1}{\tilde{\beta}_j} \left(\tilde{\beta}_j + \sum_{k=j+1}^{n} |b_{jk}|\right) = \frac{b_{jj}}{\tilde{\beta}_j}. \tag{2.11}$$

By (2.10) and (2.11), we have

$$\sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}|\right) \leq \sum_{i=1}^{n} \left(\frac{n-1}{\min\{\tilde{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j}\right). \tag{2.12}$$

Moreover, for $j = 1, 2, \ldots, n-1$,

$$\frac{b_{jj}}{\bar{\beta}_j} = \prod_{j=1}^{i-1} \frac{b_{jj} - \sum\limits_{k=j+1}^{n} |b_{jk}| l_j(B^+) + \sum\limits_{k=j+1}^{n} |b_{jk}| l_j(B^+)}{\bar{\beta}_j}$$
$$= \frac{\bar{\beta}_j + \sum\limits_{k=j+1}^{n} |b_{jk}| l_j(B^+)}{\bar{\beta}_j}$$

Electronic Journal of Linear Algebra ISSN 1081-3810
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$$= 1 + \frac{\sum\limits_{k=j+1}^{n} |b_{jk}| l_j(B^+)}{\bar{\beta}_j}$$
$$\leq 1 + \frac{\sum\limits_{k=j+1}^{n} |b_{jk}|}{\bar{\beta}_j},$$

this implies

$$\sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\bar{\beta}_j} \leq \sum_{i=1}^{n} \frac{n-1}{\min\left\{\bar{\beta}_i, 1\right\}} \prod_{j=1}^{i-1} \left(1 + \frac{1}{\bar{\beta}_j} \sum_{k=j+1}^{n} |b_{jk}|\right). \tag{2.13}$$

The conclusion follows from (2.12)and (2.13). ☐

EXAMPLE 2.6. Consider the family of B-matrices in [15]:

$$M_k = \begin{bmatrix} 1.5 & 0.5 & 0.4 & 0.5 \\ -0.1 & 1.7 & 0.7 & 0.6 \\ 0.8 & -0.1\frac{k}{k+1} & 1.8 & 0.7 \\ 0 & 0.7 & 0.8 & 1.8 \end{bmatrix},$$

where $k \geq 1$. Then $M_k = B_k^+ + C_k$, where

$$B_k^+ = \begin{bmatrix} 1 & 0 & -0.1 & 0 \\ -0.8 & 1 & 0 & -0.1 \\ 0 & -0.1\frac{k}{k+1} - 0.8 & 1 & -0.1 \\ -0.8 & -0.1 & 0 & 1 \end{bmatrix}.$$

By Theorem 1.1 (Theorem 2.2 in [10]), we have

$$\max_{d \in [0,1]^4} ||(I - D + DM_k)^{-1}||_\infty \leq \frac{4-1}{\min\{\beta, 1\}} = 30(k+1).$$

It is obvious that

$$30(k+1) \to +\infty \quad \text{when} \quad k \to +\infty.$$

By Theorem 1.3, we have

$$\max_{d \in [0,1]^4} ||(I - D + DM_k)^{-1}||_\infty \leq \sum_{i=1}^{4} \left(\frac{3}{\min\{\tilde{\beta}_i, 1\}} \prod_{j=1}^{i-1} \frac{b_{jj}}{\tilde{\beta}_j} \right) \approx 15.2675.$$

By Theorem 1.2, we have

$$\max_{d \in [0,1]^4} ||(I - D + DM_k)^{-1}||_\infty \leq \frac{2.97(90k+91)(190k+192) + 6.24(100k+101)^2}{0.99\,(90k+91)^2},$$

Electronic Journal of Linear Algebra ISSN 1081-3810
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Volume 31, pp. 476-484, July 2016

and

$$\frac{2.97(90k+91)(190k+192)+6.24(100k+101)^2}{0.99\,(90k+91)^2} < 15.2675, \quad \text{for any } k \geq 1.$$

By Theorem 2.4, we have

$$\max_{d\in[0,1]^4} ||(I-D+DM_k)^{-1}||_\infty \leq \frac{2.97(90k+91)(190k+191)+5.97(100k+100)^2}{0.99\,(90k+91)^2},$$

and

$$\begin{aligned}&\frac{2.97(90k+91)(190k+191)+5.97(100k+100)^2}{0.99\,(90k+91)^2}\\ &< \frac{2.97(90k+91)(190k+192)+6.24(100k+101)^2}{0.99\,(90k+91)^2}.\end{aligned}$$

In particular, when $k=1$,

$$\frac{2.97(90k+91)(190k+191)+5.97(100k+100)^2}{0.99\,(90k+91)^2} \approx 13.6777,$$

and

$$\frac{2.97(90k+91)(190k+192)+6.24(100k+101)^2}{0.99\,(90k+91)^2} \approx 14.1044.$$

When $k=2$,

$$\frac{2.97(90k+91)(190k+191)+5.97(100k+100)^2}{0.99\,(90k+91)^2} \approx 13.7110,$$

and

$$\frac{2.97(90k+91)(190k+192)+6.24(100k+101)^2}{0.99\,(90k+91)^2} \approx 14.1079.$$

In these two cases, the bounds in (1.5) are equal to 60 ($k=1$) and 90 ($k=2$), respectively. This example shows that the bound in Theorem 2.4 is sharper than those in Theorems 1.1, 1.2 and 1.3.

3. Conclusions. In this paper, we give a new bound for $\max_{d\in[0,1]^n} ||(I-D+DM)^{-1}||_\infty$ when M is a B-matrix, and show that it improves the bound of Theorem 2.2 of [10] in some cases, and that it is always sharper than those of Theorem 4 of [15] and of Corollary 1 of [16].

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