THE COMBINATORIAL INVERSE EIGENVALUE PROBLEM II: ALL CASES FOR SMALL GRAPHS

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Abstract. Let $G$ be a simple undirected graph on $n$ vertices and let $S(G)$ be the class of real symmetric $n \times n$ matrices whose nonzero off-diagonal entries correspond to the edges of $G$. Given $2n-1$ real numbers $\lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \geq \cdots \geq \lambda_{n-1} \geq \mu_{n-1} \geq \lambda_n$, and a vertex $v$ of $G$, the question is addressed of whether or not there exists $A \in S(G)$ with eigenvalues $\lambda_1, \ldots, \lambda_n$ such that $A(v)$ has eigenvalues $\mu_1, \ldots, \mu_{n-1}$, where $A(v)$ denotes the matrix with $v$th row and column deleted. A complete solution can be given for the path on $n$ vertices with $v$ a pendant vertex and also for the star on $n$ vertices with $v$ the dominating vertex. The main result is a complete solution to this "$\lambda, \mu$" problem for all connected graphs on 4 vertices.

Key words. Interlacing inequalities, Inverse eigenvalue problem, Symmetric matrix.

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