ON THE INVERSE OF A CLASS OF BIPARTITE GRAPHS WITH UNIQUE PERFECT MATCHINGS

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Abstract. Let $G$ be a simple, undirected graph and $G_w$ be the positive weighted graph obtained from $G$ by giving weights to its edges using the positive weight function $w$. A weighted graph $G_w$ is said to be nonsingular if its adjacency matrix $A(G_w)$ is nonsingular. Let $\mathcal{G}$ denote the class of connected, unweighted, bipartite graphs $G$ with a unique perfect matching $M$ such that $G/M$ (the graph obtained by contracting the matching edges in $G$) is bipartite. Similarly, let $\mathcal{G}_w$ denote the class of connected, weighted, bipartite graphs $G_w$ with a unique perfect matching such that the underlying unweighted graph $G \in \mathcal{G}$. These graphs are known to be nonsingular. In (*Inverses of trees*, Combinatorica, 5(1):33–39, 1985), Godsil showed that if $G \in \mathcal{G}$, then $A(G)^{-1}$ is signature similar to a nonnegative matrix, that is, there exists a diagonal matrix $D$ with diagonal entries $\pm 1$ such that $DA(G)^{-1}D$ is nonnegative. The graph associated to the matrix $DA(G)^{-1}D$ is called the inverse of $G$ and it is denoted by $G^+$. The graph $G^+$ is an undirected, weighted, connected, bipartite graph with a unique perfect matching. Notice that unweighted trees which are nonsingular are contained inside the class $\mathcal{G}$.

In (*On reciprocal eigenvalue property of weighted trees*, Linear Algebra and its Applications, 438:3817–3828, 2013), Neumann and Pati have characterized graphs that occur as inverses of nonsingular, unweighted trees. We generalize this result and constructively characterize the class of weighted graphs which can occur as the inverse of any graph in $\mathcal{G}$. We also show that for a graph $G \in \mathcal{G}$, the inverse $G^+ \in \mathcal{G}$ if and only if $G \cong G^+$ (isomorphic).

A weighted graph $G_w$ is said to have the property R if for each eigenvalue $\lambda$ of $A(G_w)$, $1/\lambda$ is also an eigenvalue of $A(G_w)$. If further, the multiplicity of $\lambda$ and $1/\lambda$ are the same, then $G_w$ is said to have property SR. A characterization of the class of nonsingular, weighted trees $T_w$ with at least 8 vertices that have property R was given in (*On reciprocal eigenvalue property of weighted trees*, Linear Algebra and its Applications, 438:3817–3828, 2013) under some restriction on the weights. It is natural to ask for such a characterization for the whole of $\mathcal{G}_w$, possibly with some weaker restrictions on the weights. We supply such a characterization. In particular, for trees it settles an open problem raised in (*On reciprocal eigenvalue property of weighted trees*, Linear Algebra and its Applications, 438:3817–3828, 2013).

Key words. Adjacency matrix, Inverse graph, Property R, Property SR, Corona, Weighted graph.

AMS subject classifications. 15A18, 05C50

*Received by the editors on January 6, 2015. Accepted for publication on August 27, 2015. Handling Editor: Steve Kirkland.
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