SPARSE SPECTRALLY ARBITRARY PATTERNS∗

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Abstract. We explore combinatorial matrix patterns of order n for which some matrix entries are necessarily nonzero, some entries are zero, and some are arbitrary. In particular, we are interested in when the pattern allows any monic characteristic polynomial with real coefficients, that is, when the pattern is spectrally arbitrary. We describe some order n patterns that are spectrally arbitrary. We show that each superpattern of a sparse companion matrix pattern is spectrally arbitrary. We determine all the minimal spectrally arbitrary patterns of order 2 and 3. Finally, we demonstrate that there exist spectrally arbitrary patterns for which the nilpotent-Jacobian method fails.

Key words. Companion matrix pattern, Spectrally arbitrary pattern, Nilpotent-Jacobian method, Nonzero pattern.

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