Properties of First Eigenvectors and First Eigenvalues of Nonsingular Weighted Directed Graphs∗

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Abstract. The class of nonsingular connected weighted directed graphs with an unweighted undirected branch is considered in this article. This paper investigates the monotonicity properties of the first eigenvectors of such graphs along certain paths. The paper describes how the first eigenvalue of such graphs changes under some perturbation. It is shown that replacing a branch which is a tree by a path on the same number of vertices will not increase the first eigenvalue, while replacing the tree by a star on the same number of vertices will not decrease the first eigenvalue. As an application the paper characterizes the graphs minimizing the first eigenvalue over certain classes of such graphs.

Key words. Laplacian matrix, Weighted directed graph, First eigenvalue, First eigenvector.

AMS subject classifications. 05C50, 05C05, 15A18.