REVERSE JENSEN-MERCER TYPE OPERATOR INEQUALITIES∗

EHSAN ANJIDANI† AND MOHAMMAD REZA CHANGALVAIY†

Abstract. Let $A$ be a selfadjoint operator on a Hilbert space $\mathcal{H}$ with spectrum in an interval $[a, b]$ and $\phi : B(\mathcal{H}) \to B(\mathcal{K})$ be a unital positive linear map, where $\mathcal{K}$ is also a Hilbert space. Let $m, M \in J$ with $m < M$ such that either $m + M \leq a + b$ and $A \leq m$, or $m + M \geq a + b$ and $A \geq M$. If $f$ is convex on $J$, then the inequality

$$f(m_{1\mathcal{K}} + M_{1\mathcal{K}} - \phi(A)) \geq f(m)_{1\mathcal{K}} + f(M)_{1\mathcal{K}} - \phi(f(A)),$$

is proved. A variant of this inequality is established for superquadratic functions. The results obtained are used to prove some comparison inequalities between operators of power and quasi-arithmetic mean’s type.

Key words. Jensen-Mercer operator inequality, Convex function, Superquadratic function, Operator power mean, Operator quasi-arithmetic mean.

AMS subject classifications. 47A63, 47A64, 15A60.

∗Received by the editors on July 9, 2015. Accepted for publication on January 13, 2016. Handling Editor: Bryan L. Shader.
†Department of Mathematics, University of Neyshabur, PO Box 91136-899, Neyshabur, Iran (anjidani@neyshabur.ac.ir, mr.ch1392@gmail.com).