INVERTIBLE AND REGULAR COMPLETIONS
OF OPERATOR MATRICES

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Abstract. In this paper, for given operators $A \in \mathcal{B} (\mathcal{X})$ and $B \in \mathcal{B} (\mathcal{Y})$, the set of all $C \in \mathcal{B} (\mathcal{Y}, \mathcal{X})$ such that the operator matrix $M_C = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$ is injective, invertible, left invertible and right invertible, is described. Answers to some open questions are given. Also, in the case when $A$ and $B$ are relatively regular operators, the set of all $C \in \mathcal{B} (\mathcal{Y}, \mathcal{X})$ such that $M_C$ is regular is described. In addition, a necessary and a sufficient conditions are given for $M_C$ to be regular with the inner inverse of a certain given form.

Key words. Invertibility, Operator matrix, Regularity, Inner inverse.

AMS subject classifications. 47A10, 47A53, 47A55.

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