Abstract. Lagrangian subspaces are linear subspaces that appear naturally in control theory applications, and especially in the context of algebraic Riccati equations. In this paper, a class of semidefinite Lagrangian subspaces is introduced and it is shown that these subspaces can be represented by a subset \( I \subseteq \{1, 2, \ldots, n\} \) and a Hermitian matrix \( X \in \mathbb{C}^{n \times n} \) with the property that the submatrix \( X_{II} \) is negative semidefinite and the submatrix \( X_{I^c I^c} \) is positive semidefinite. A matrix \( X \) with these definiteness properties is called \( I \)-semidefinite and it is a generalization of a quasidefinite matrix. Under mild hypotheses which hold true in most applications, the Lagrangian subspace associated to the stabilizing solution of an algebraic Riccati equation is semidefinite, and in addition it is shown that there is a bijection between Hamiltonian and symplectic pencils and semidefinite Lagrangian subspaces; hence, this structure is ubiquitous in control theory. The (symmetric) principal pivot transform (PPT) is a map used by Mehrmann and Poloni [V. Mehrmann and F. Poloni. Doubling algorithms with permuted Lagrangian graph bases. SIAM J. Matrix Anal. Appl., 33:780–805, 2012.] to convert between two different pairs \( (I, X) \) and \( (J, X') \) representing the same Lagrangian subspace. For a semidefinite Lagrangian subspace, it is proven that the symmetric PPT of an \( I \)-semidefinite matrix \( X \) is a \( J \)-semidefinite matrix \( X' \), and an implementation of the transformation \( X \mapsto X' \) that both makes use of the definiteness properties of \( X \) and guarantees the definiteness of the submatrices of \( X' \) in finite arithmetic is derived. The resulting formulas are used to obtain a semidefiniteness-preserving version of an optimization algorithm introduced by Mehrmann and Poloni to compute a pair \( (I_{\text{opt}}, X_{\text{opt}}) \) with \( M_{\text{opt}} = \max_{i,j} |(X_{\text{opt}})_{ij}| \) as small as possible. Using semidefiniteness allows one to obtain a stronger inequality on \( M \) with respect to the general case.

Key words. Lagrangian subspace, Symplectic pencil, Hamiltonian pencil, Principal pivot transform, Quasidefinite matrix, Riccati matrix, Graph matrix.

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