EXTREMAL GRAPHS FOR THE SUM OF THE TWO LARGEST SIGNLESS LAPLACIAN EIGENVALUES

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Abstract. Let \( G \) be a simple graph on \( n \) vertices and \( e(G) \) edges. Consider the signless Laplacian, \( Q(G) = D + A \), where \( A \) is the adjacency matrix and \( D \) is the diagonal matrix of the vertices degree of \( G \). Let \( q_1(G) \) and \( q_2(G) \) be the first and the second largest eigenvalues of \( Q(G) \), respectively, and denote by \( S_n^+ \) the star graph with an additional edge. It is proved that inequality \( q_1(G) + q_2(G) \leq e(G) + 3 \) is tighter for the graph \( S_n^+ \) among all firefly graphs and also tighter to \( S_n^+ \) than to the graphs \( K_k \lor K_{n-k} \) recently presented by Ashraf, Omidi and Tayfeh-Rezaie. Also, it is conjectured that \( S_n^+ \) minimizes \( f(G) = e(G) - q_1(G) - q_2(G) \) among all graphs \( G \) on \( n \) vertices.

Key words. Signless Laplacian, Sum of eigenvalues, Extremal graphs.

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