AN INVERSE EIGENPROBLEM FOR GENERALIZED REFLEXIVE MATRICES WITH NORMAL \( \{k+1\}\)-POTENCIES

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Abstract. Let \( P, Q \in \mathbb{C}^{n \times n} \) be two normal \( \{k+1\}\)-potent matrices, i.e., \( PP^* = P^*P, \ P^{k+1} = P \), \( QQ^* = Q^*Q, \ Q^{k+1} = Q \), \( k \in \mathbb{N} \). A matrix \( A \in \mathbb{C}^{n \times n} \) is referred to as generalized reflexive with two normal \( \{k+1\}\)-potent matrices \( P \) and \( Q \) if and only if \( A = PAQ \). The set of all \( n \times n \) generalized reflexive matrices which rely on the matrices \( P \) and \( Q \) is denoted by \( \mathcal{GR}^{n \times n}(P,Q) \). The left and right inverse eigenproblem of such matrices ask from us to find a matrix \( A \in \mathcal{GR}^{n \times n}(P,Q) \) containing a given part of left and right eigenvalues and corresponding left and right eigenvectors. In this paper, first necessary and sufficient conditions such that the problem is solvable are obtained. A general representation of the solution is presented. Then an expression of the solution for the optimal Frobenius norm approximation problem is exploited. A stability analysis of the optimal approximate solution, which has scarcely been considered in existing literature, is also developed.

Key words. Left and right inverse eigenproblem, Optimal approximation problem, Generalized reflexive matrix, Moore-Penrose generalized inverse, \( \{k+1\}\)-Potent matrix.

AMS subject classifications. 65F18, 15A51, 15A18, 15A12.

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