Evaluation of Collocated Measurements of Radar Reflectivity and Particle Sizes in Ice Clouds

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Evaluation of Collocated Measurements of Radar Reflectivity and Particle Sizes in Ice Clouds

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ABSTRACT

Measured 94-GHz reflectivity in midlevel, stratiform ice clouds was compared with reflectivity calculated from size distributions determined with a particle imaging probe. The radar and the particle probe were carried on the same aircraft, the Wyoming King Air, ensuring close spatial correspondence between the two measurements. Good overall agreement was found within the range from $-18$ to $+16$ dBZ, but there is an important degree of scatter in the results. Two different assumptions about particle density led to calculated values that bracket the observations. The agreement found for reflectivity supports the use of the data for establishing relationships between the measured reflectivity and ice water content and between precipitation rate and reflectivity. The resulting equation for ice water content (IWC vs $Z$) agrees with the results of Liu and Illingworth within a factor of 2 over the range of overlap between the two datasets. The equation here reported for precipitation rate (PR vs $Z$) has a shallower slope in the power-law relationship than that reported by Matrosov as a consequence of sampling particles of greater densities. Because the radar and the particle probe were collocated on the same platform, errors arising from differences in sampling locations and volumes were minimized. Therefore it is concluded that the roughly factor-of-10 spread in IWC and in PR for given $Z$ is, primarily, a result of variations in ice crystal shape and density. Retrievals of IWC and PR from cloud radar data can be expected to have that level of uncertainty.

1. Introduction

Because cloud properties are important determinants in climate models—in precipitation forecasts and in a number of other areas—efforts to diagnose such properties as liquid or ice water content and particle size distributions have been receiving intensive attention from both the measurement and modeling perspectives (e.g., Ramanathan et al. 1989; Stephens 2005). Measurements by aircraftborne instruments, and by remote sensing from the ground and from satellites, are the principal sources of cloud composition data, and it is of considerable importance to establish the best possible correspondence between the two types of measurements. This paper is a contribution to that effort.

The relationship sought in this paper is between in situ observations of clouds predominantly composed of ice crystals and radar reflectivity at 94 GHz. Radars operating at 94 GHz are becoming of greater importance with the deployment of CloudSat (Brown et al. 1995; Stephens et al. 2002; Marchand et al. 2008; Protat et al. 2009) and the installation of ground-based units (Hogan and Illingworth 2003). The results here reported should enhance the utility of the data derived from these radars and others operating at the same frequency. Some aspects of the results may have more general applicability.

Previous work relating millimeter-wavelength radar observations to cloud composition is extensive (e.g., Liu and Illingworth 2000; Lhermitte 2002; Frisch et al. 2002; Okamoto 2002; Wang et al. 2005; Sato and Okamoto 2006; Protat et al. 2007, 2009; Delanoë et al. 2007; Matrosov et al. 2008) and includes both observational and theoretical approaches. Data used in most of these analyses combined radar (and lidar) measurements from the ground, or from space, with in-cloud observations from aircraft. Here we report results based on radar and in situ measurements collected from the same aircraft. The main benefit from that arrangement is that, while the radar sample volume is still about $10^5$ times that of the in situ observations, that ratio is considerable better than the $10^{10}$ or greater ratio typical with the use of ground-based or space-based radars. The smaller
sample volume of the airborne radar lessens the degrada-
dation in data quality that results from averaging re-
fectivity over large inhomogeneous cloud volumes, given
that reflectivity is a highly nonlinear measure of hyd-
meteors sizes.

This paper reports results obtained with the 94-GHz
Wyoming Cloud Radar (WCR) and the Particle Mea-
suring Systems, Inc. (PMS) two-dimensional cloud-
imaging (2D-C) probe carried on the Wyoming King Air
(WKA) aircraft during studies of winter orographic
clouds. After it is established that the measured radar
reflectivity and the value calculated from probe data are
well correlated, the data are used to derive relationships
between reflectivity and ice water content (IWC) and
between reflectivity and precipitation rate (PR). The
derived relationships, and quantification of the range of
inherent variability associated with them, may prove to
be helpful in the interpretation of radar measurements
for cloud studies and in their application in cloud and
climate models.

2. Data description

a. Instrumentation

The WKA research aircraft used in these studies was
equipped with an array of instruments for in situ mea-
surement of thermodynamic and cloud parameters
and with the WCR (for a description of the aircraft pro-
gram, see online at http://flights.uwyo.edu/wcr). The
94-GHz WCR was operated in a three-beam configura-
tion consisting of a single upward-looking beam and two
downward-looking beams (one directed at nadir and the
other slanted ~30° forward). In this study, data only from
the up and down beams are used. Radar reflectivity and
particle velocity profiles were recorded approximately
every 3 m of flight distance. Each profile extended out-
ward from ~100 m above and 100 m below the WKA to
3 km, with 30-m spacing of range gates. For more details
about the WCR see Pazmany et al. (1994) or online
(http://flights.uwyo.edu/wcr). The absolute accuracy of
the radar measurements, based on calibrations with a
corner reflector, is estimated to be about ±2.5 dBZ.
The stability (precision) observed over weeks of usage
is estimated, from surface returns and from repeated
calibrations, to be ±1 dBZ (S. Haimov 2010, personal
communication).

For observations on cloud composition, the major
data source for this paper was the PMS 2D-C probe
(Knollenberg 1970). Complementary measurements were
of thermodynamic and air motion parameters. The 2D-C
probe provides ice crystal concentration and size data.
The resolution of the probe is 25 μm, the array width
is 800 μm, and volume sampling rate is roughly 5 ×
10⁻³ m³ s⁻¹. Sizes were recorded in 20 bins of pro-
gressively increasing width from 0–50 to 600–700 μm.
Details of the data-processing routine are given in the
appendix. Problems inherent to deriving size and shape
information from shadow images of the particles lead to
imprecision and uncertainty that are difficult to deter-
mine. Extensive literature on evaluating probe perfor-
mance (e.g., Heymsfield and Baumgardner 1985; Gayet
et al. 1993; Korolev et al. 1998; Korolev 2007) shows that
in many cases the probe data provide adequate mea-
urements but that in some situations the probe is prone
to errors due to particle shattering at the probe tips and
also that it provides inadequate sampling of large par-
ticles of low concentration. In clouds that are similar
to the ones examined in this work, Cooper and Vali
(1981) and Cooper and Saunders (1980) have shown
agreement between ice crystal concentrations measured
by the 2D-C probe and impaction samples taken in a
deaccelerator.

For about one-half of the total dataset (from the Na-
tional Aeronautics and Space Administration NASA06
project described in the next section), a 2D-P (preci-
pitation) probe was also installed on the WKA. The
resolution of this probe is nearly 10 times as large, the
array width is 0.64 cm, and it has a sampling rate that
is roughly 10 times as great. For the data used here,
reflectivity derived from the 2D-C and 2D-P probes was
found to be in good agreement. The difference between
mean values from the two probes ranged between −0.5
and 2.5 dBZ for 71% of the cases. For reflectivities
above 0 dBZ, the linear fit to reflectivity-decibel values
has a slope of 1.2 and an offset of −0.94, and the cor-
relation coefficient is 0.96. If lower reflectivities are also
included, the agreement is degraded by large numbers
of points for which the 2D-P values are lower than
the 2D-C values. This can be ascribed to an inadequate
sampling of particles of submillimeter sizes with the
2D-P probe. In light of these results—to be specific, the
assurance they provide that there was no significant un-
dersampling of large particles—the following analyses
were based on the 2D-C data. Note also that because of
Mic effects at the radar wavelength used in this work
(3 mm) the scattering cross sections of particles of larger
than about 1 mm are greatly decreased in comparison
with the sixth power of the diameter that is valid below
that size. This reduces very significantly the importance
of particles that may be undersampled with the 2D-C
probe.

b. Data sources

Data are used in this paper from WKA flights made
during two winter projects labeled NASA06 and
“WAICO08.” The former took place during January and February of 2006, and the latter occurred from January through March of 2008. The flights were made over the Medicine Bow mountains of southeastern Wyoming; the center point of the flight tracks was roughly 41.37°N, 106.33°W. The field projects were not specifically designed for the purposes of this paper, but nonetheless the data are suitable for these analyses.

Data used here are from winter orographic clouds that can be classified as stratiform with weak embedded convection. Cloud depths ranged from 2 to 5 km; their horizontal extent was over 100 km in all cases. Sampling was conducted along level flight legs. An example of this type of cloud is illustrated with the radar image shown in the top panel of Fig. 1. Most of the clouds selected for analysis were composed entirely of ice crystals at flight level. Supercooled liquid was present at flight level in less than 10% of the total dataset and liquid water content never exceeded 0.3 g m⁻³. This assured that the contribution of cloud droplets to the measured radar reflectivity was negligible.

The flight segments (time periods) to be used in the analysis were selected to represent relatively uniform conditions. Criteria for this included the continuous presence of cloud and constant flight altitude, variations in ice particle concentration not to exceed a factor of 2, and small and slow variations in radar reflectivity. Application of these criteria in a visual inspection of graphic data displays led to the selection of 63 time segments from six different flights of NASA06 and 65 time segments from nine different flights of WAICO08. The selected time intervals vary from 16 to 240 s in flight duration, corresponding to roughly 1.6–24 km in horizontal extent. The average length of selected segments is 3.5 km. The total numbers of data points (1-s samples) used in this study are 3295 and 2645 from NASA06 and WAICO08, respectively. The temperatures range from −26° to −5° C in the NASA06 samples and from −37° to −15°C for WAICO08. Ice concentrations range from <1 to >50 L⁻¹ in NASA06 and up to 80 L⁻¹ in WAICO08. The distribution of temperature and of ice concentrations within the sample set is summarized in

**Fig. 1.** (top) Radar reflectivity in a vertical section through a deep mountain cloud overlying two mountain ranges. The aircraft flight level was at 5.2 km; the vertical section was obtained by combining data from the upward- and downward-pointing antennas. (bottom) The interpolated reflectivity at flight level (continuous line), the reflectivity calculated from the 2D-C image data (squares) during selected segments, and the concentration of ice particles (diamonds; right-hand scale) during those segments. The data are for the period 1335:25–1359:30 UTC from a flight conducted on 31 Jan 2006. The distance scale is with respect to the peak of the Medicine Bow range. The flight line was roughly west to east.
Table 1. Most of the data come from temperatures between \(-10^\circ\) and \(-30^\circ\)C, and with ice concentrations of <30 L\(^{-1}\).

An illustration of the type of data that are addressed in this paper is given in Fig. 1 for a pass through a deep cloud layer. The flight was made on 31 January 2006 over the Sierra Madre and Medicine Bow ranges of southeastern Wyoming. The top panel shows radar reflectivity in a vertical section. The white line at 5.2 km shows the aircraft flight track. The bottom panel in Fig. 1 has observed reflectivity values from an interpolation of measured values to the flight level (continuous line), the values calculated from the 2D-C image data for selected intervals (squares), and the ice particle concentrations during those intervals (diamonds). Sample images for this pass from the 2D-C probe are given in Fig. 2, illustrating that most particles were irregular in shape. Size distributions for two intervals from this pass are shown in Fig. 3. The main difference is seen in the number of smaller particles so that the calculated reflectivity is nearly the same for the two segments despite a factor-of-3 difference in particle concentrations.

c. Data processing

Processing of the 2D-C image data consisted of several steps: 1) recognizable artifacts are rejected; 2) an elliptical shape is fit to each accepted image and the radius of a sphere equal in volume to the prolate ellipsoid is calculated; 3) mass and fall velocity are estimated; 4) the backscatter cross section of the particle is determined using Mie theory; 5) by summing the contributions of all particles, the IWC (g m\(^{-3}\)), PR (mm h\(^{-1}\)), and radar backscatter per unit volume (mm\(^2\) m\(^{-3}\)) are calculated at 1-Hz frequency (typical sampled volume of 0.005 m\(^3\)). Steps 3–5 are carried out with two different assumptions for particle density. Details of the first two steps are given in the appendix.

To assess the relative importance of larger particles, the mass contributed by particles that are smaller than 1 mm was extracted in a separate variable. The 1-Hz data points for which this contribution exceeded 80% are denoted with “mf.” This additional stratification leads to four data cuts: “d1” and “d3” for the two density values and “d1 mf” and “d3 mf” for the small-particle classes. The fraction of 1-Hz samples in the mf subsets is 74% for d1 and 75% for d3.

1) MASS OF ICE AND PRECIPITATION RATE

Direct measurements of IWC were not available for these studies. Thus, particle density had to be assumed

1 Subscripts 1, 3, 1mf, and 3mf are used to label values pertaining to the four stratifications. When not referring to any specific subset, subscripts “calc” and “obs” are used to distinguish between derived and measured values.
on the basis of similarity to published data. In principle this could be done on a particle-by-particle basis by relying on the 2D images to assess crystal type; the imprecision of image classification coupled with the limited extent of published observations by crystal type undermines the advantages of this approach, however. Therefore, we decided to use the same density-versus-size functions throughout. Visual assessment of the particle images in our data (cf. Fig. 2) indicates that the sampled particles were predominantly irregularly shaped graupel. Density data for this crystal type are available from measurements taken at the Elk Mountain Observatory by Zikmunda and Vali [1972 (ZV72)] and taken from the Cascade Mountains by Locatelli and Hobbs [1974 (LH74)]. The advantage of the Elk Mountain data is that they were taken at 3000-m elevation in the same mountain range as where the aircraft data for this paper were collected. An important shortcoming of both datasets is that the size range covered by the measurements barely extends below 0.1 cm, however.

In more recent publications, particle density functions are derived from aircraft measurements of particle size distributions combined with coincident measurements of the IWC or radar reflectivity. Most of these observations are from cirrus. The results of Brown and Francis [1995 (BF95)] for aggregates of unrimed crystals are widely used. Their results were in good agreement with the LH74 particle-by-particle observations for this crystal type. Liu and Illingworth [2000 (LI00)] presented further evidence for the applicability of those results to ice clouds somewhat more broadly defined than just cirrus. Heymsfield et al. [2010 (H10)] present a good discussion of factors that influence interpretation of the LH74 and BF95 data and conclude that their own large dataset justifies a result that is only slightly different from those, again focusing on a broad range of ice clouds.

The dependence of particle density on size from these various sources is depicted in Fig. 4 along with two choices of the function \( \rho_1 \) and \( \rho_3 \) used in this paper. As shown by the ZV72 and LH74 lines, these two ground-based measurements converge at the upper end of the range of data and diverge importantly at small sizes. The higher density of small graupel in the ZV72 data may be due to the higher-altitude and more-continental cloud types characteristic of their data. Nonetheless, even the higher densities in our assumed function \( \rho_3 \) represent a compromise between the ZV72 and LH74 data.

The velocity functions used in this work (Fig. 5) also represent a compromise between the ZV72 and LH74 datasets. The ratio of fall velocities for given particle sizes is equal to the density ratios to the power 0.8, on the basis of the Reynolds number–versus–Best number analysis of Mitchell (1996). The choice of the exponent 0.8 is in roughly in the middle of the range indicated in that paper. The two sets of density, mass, and fall-velocity functions used in this work, designated as d1 and d3, are listed below. Although these forms of exponential functions are not dimensionally invariant (they should have \( D \) normalized to a unit value), we stay with the following conventional usage—

1) case d1: \( \rho_1 = 0.1D^{-0.33}, \quad M_1 = 0.052D^{2.67}, \quad V_1 = 3.6D^{0.5} \)
2) case d3: \( \rho_3 = 0.1D^{-0.45}, \quad M_3 = 0.052D^{2.55}, \quad V_3 = 3.6D^{0.4} \)

Units in these equations follow common usage: \( \rho \) is in grams per centimeter cubed, \( D \) is in centimeters, \( M \) is in grams, and \( V \) is in meters per second. In the d3 case, density was assigned a constant value of 0.8 for \( D < 0.0125 \) cm, and hence \( M_3 \) is proportional to the cube of the diameter. As can be seen in Fig. 4, these functions are not far from the Elk Mountain (ZV72) and Cascade Mountain (LH74) datasets. Both sets of measurements were performed at 2–3-km altitudes; therefore the equations given above are also valid for similar altitudes. No further adjustments were made to account for the actual altitudes of the flights, because that change would have been negligible in comparison with the uncertainties of the assumed fall-velocity equations.
Although the degree of validity of the foregoing assumptions clearly is not uniform throughout the dataset, using a single set of assumptions was adopted to avoid stratifications that would not be fully defensible with the data quality available and to determine the degree of usefulness that can be achieved without further elaboration. Deviations from the assumed relationships can be expected, principally because of variations in the size distributions of the sampled crystals, their growth habits, and the extent of riming. These are difficult characteristics to reduce to a limited set of categories. In the majority of the cases, ice particles were irregular in shape, probably because of varied temperature growth regimes and riming, but a small number of cases with pristine crystals were also included and will be discussed later on.

2) REFLECTIVITY

The refractive index for ice particles as a function of density was calculated on the basis of the Maxwell-Garnet formula (Bohren and Battan 1980) for the two density functions $\rho_1$ and $\rho_3$. With the resulting two sets of refractive indices, two backscatter cross sections were calculated using the Mie formulas given by Ulaby et al. (1981, p. 292) for sizes corresponding to the bin centers of the frequency distributions of particle sizes. Application of the Mie equations to complex crystal shapes is an approximation that has definite limitations, but application of more precise numerical methods would have been prohibitive in computational burden and would still suffer from the idealizations involved in constructing geometric shapes to describe the crystals. Figure 6 shows the values obtained for the refractive index and for the scattering cross section as functions of size for the $d_1$ density function. Summation over the size distribution yielded two values $Z_1$ and $Z_3$ for each second of data; collectively the two are referred to as $Z_{\text{cal}}$.

For the comparison of in situ probe data with radar measurements, one would ideally like to have observed radar reflectivity right along the flight line. The radar dead zone limits measurements to 100 m above and below the aircraft, however. To overcome this problem an interpolation is performed between reflectivities above and below the flight line. Reflectivity values from the first five range gates above and below the flight level are used to fit a line across the total 500-m range of measured values for each radar profile and to extract from that line the interpolated reflectivity at flight level. This method was chosen because it is less influenced by noise than a simple averaging of the two closest range gates would be while still allowing for real variations in

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2 Reflectivity is expressed throughout this paper as water equivalent reflectivity even when the customary subscript "e" is not indicated. In conformity with general practice, the symbol $Z$ is used for reflectivity both when given in units of millimeters to the sixth power per cubic meter or as reflectivity decibels (dBZ); the distinction is specified whenever it is needed.
the vertical direction. Typical variations of reflectivity in the vertical direction (the slope of the fitted line) are between $2\pm 2$ dB Z per 100 m in altitude. The derived radar reflectivity at flight level is resampled at 1 Hz (average of roughly 30 profiles) to match the data rate of the reflectivity values calculated from the in situ probe data.

3. Results

a. Measured versus calculated reflectivity

The radar-observed reflectivity values $Z_{\text{obs}}$ are first compared with values calculated from the particle imaging data ($Z_1$ and $Z_3$) using correlation analysis. This is done in two slightly different ways: 1) using 1-s averages of $Z_{\text{obs}}$ and 1-s cumulative counts of size-sorted particle data and 2) using 5-, 50-, and 95-percentile values of both $Z_{\text{calc}}$ and $Z_{\text{obs}}$ for each selected flight segment. Because the time segments were selected to be relatively uniform, not much difference is expected between these two approaches. The first method results in more complete depictions of the data, but the second method allows variations within time segments to be displayed and to readily associate possible outliers or other notable groupings of points with specific segments.

Figure 7 shows scatterplots of the measured and calculated reflectivities from the complete dataset at 1-Hz resolution. The top panel in Fig. 7 is for the $d1$ density assumption, and the bottom panel is for $d3$; in both cases the gray points designate the $mf$ subsets. Solid lines show the best-fit linear models to the dBZ values (slopes $m_1 = 0.86$ and $m_3 = 0.83$; intercepts $C_1 = 0.51$ and $C_3 = 3.6$; correlation coefficients $r_1 = r_3 = 0.94$), and the dashed line shows a 1:1 relationship. The dash–dotted lines bracket 90% of the points. This range was determined by grouping points within 5-dBZ intervals of $Z_{\text{obs}}$ and determining the limits that excluded 5% of the $Z_{\text{calc}}$ values at both the high and low ends. Linear models were fit to these limits with weighting of each interval by the number of 1-Hz points it contained. For the upper limits $m_1 = 0.74$, $m_3 = 0.70$, $C_1 = 6.0$, and $C_3 = 8.9$, and for the lower limits $m_1 = 1.03$, $m_3 = 0.98$, $C_1 = -5.4$, and $C_3 = -2.1$. The total number of 1-Hz data points in Fig. 7 is 5940, with 3295 of these coming from the NASA06 project and 2645 from the WAICO08 project. Separate correlations were also generated for the $mf$ subset. The linear fit and correlation coefficients, as well

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3 All linear fits in this paper use a method of least absolute deviations, applying the “ladfit” routine of the proprietary IDL scientific data visualization software package.
as the ranges of validity of the different stratifications of the 1-Hz data are summarized in the first section of Table 2. It also includes the parameters separately for the two projects to illustrate the degree of variation that is present in the results and to show that for each dataset separately the slopes of the best-fit lines are closer to unity than for the combined dataset. The effect of the density assumption d1 versus d3 is a roughly 3-dB difference in C, with smaller offsets for d1.

The comparison of measured and calculated values on a flight segment–by–flight segment basis is shown in Fig. 8 for the d1 density assumption. The d3 results are very similar but are displaced toward higher values by roughly 3 dBZ. In this graph, the points show the 50-percentile values and the vertical and horizontal bars on selected points extend to the 5- and 95-percentile values for the given flight segments. A total of 128 time segments are included, from 15 different flights (63 time segments from NASA06 and 65 segments from WAICO08). The solid line represents the linear regression to the medians (dBZ) ($r_1 = 0.96$, $m_1 = 0.88$, and $C_1 = -0.1$), and the dash–dotted lines bracket 90% of the 1-Hz points as in Fig. 7. The linear fit parameters for d3 are $r_3 = 0.96$, $m_3 = 0.84$, and $C_3 = 3.0$. There are no extreme outliers in Fig. 8 that would call attention to some special circumstance with particular data segments. Table 2 lists the various fit parameters for the complete dataset.

As expected, the best-fit relationships derived by the two alternative ways of data processing (1-Hz and flight segment) yield nearly identical results and similarly high correlation coefficients. Calculated values are slightly higher than a 1:1 agreement for reflectivities of less than 0 dBZ and are lower for values of greater than 0 dBZ; the difference amounts to +2 dBZ at −20 dBZ and −2 dBZ at +20 dBZ.

The two projects yield slightly different results, as seen in the first part of Table 2 for the 1-Hz data. This is a likely consequence of the somewhat different cloud types that were sampled in response to project objectives. The WAICO08 (w) set has marginally higher correlation coefficients and fit slopes that are closer to unity than has the NASA06 (n) set; the biases are larger, however. Measured reflectivities extend to lower values for w, and this can account for the higher correlations for that set.

Another measure of the agreement between $Z_{\text{calc}}$ and $Z_{\text{obs}}$ is the difference between these two values. Table 2 lists the median difference (dBZ) between the calculated and observed values $\Delta Z = Z_{\text{calc}} - Z_{\text{obs}}$. These differences can be readily converted from dBZ to the ratio of the two quantities (a 3-dBZ difference corresponds to a ratio of 2). For the n set, these median differences bracket zero, being negative for d1 and positive for d3, with magnitudes that indicate agreement between $Z_{\text{calc}}$ and $Z_{\text{obs}}$ within a factor of 1.5. For the w set the discrepancy is larger. Only the n d1 set has $\Delta Z < 0$, indicating that the general trend is for the calculated reflectivity to overestimate the observed value. In looking at the difference in more detail, it is found that $\Delta Z$ decreases with increasing $Z_{\text{obs}}$, as shown in Fig. 9, with a superimposed wave near the middle of the range. This pattern holds for both d1 and d3.
The variability seen in Fig. 7 for both $Z_{\text{calc}}$ and $Z_{\text{obs}}$ is very large. For any given value of one parameter the variation in the other is roughly of the same magnitude. Although it cannot be accomplished in a definite way, some evaluation can be made of the relative contributions of fluctuations in cloud composition and of sampling.

The main observation in that regard is that the scatter of points in Figs. 7 and 8 is very similar despite having much larger samples represented by each point in Fig. 8 than in Fig. 7. The 90% bounds determined for the 1-Hz data are also nearly valid for the segment-by-segment data; the latter do not have enough points to determine the 90% bounds reliably.

As can be seen in Fig. 8, the spread in values has two components: deviations of the points from the trend line and variations within each flight segment as indicated by the error bars (90% range) around the median values. One can note that the spread of points (median values) is

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### Table 2. Statistics of observed vs calculated reflectivities

<table>
<thead>
<tr>
<th>Dataset</th>
<th>$N$</th>
<th>$r$</th>
<th>Intercept $C$</th>
<th>Slope $m$</th>
<th>Median diff (dBZ)</th>
<th>98% range (dBZ)</th>
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<tbody>
<tr>
<td><strong>1 Hz</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 All d1</td>
<td>5940</td>
<td>0.94</td>
<td>0.51</td>
<td>0.86</td>
<td>0.03</td>
<td>-24.1 to 16.2</td>
</tr>
<tr>
<td>2 All d1 mf</td>
<td>4394</td>
<td>0.94</td>
<td>0.22</td>
<td>0.86</td>
<td>0.03</td>
<td>-25.3 to 18.1</td>
</tr>
<tr>
<td>3 All d3</td>
<td>5940</td>
<td>0.94</td>
<td>3.6</td>
<td>0.83</td>
<td>2.84</td>
<td>-20.0 to 18.7</td>
</tr>
<tr>
<td>4 All d3 mf</td>
<td>4447</td>
<td>0.94</td>
<td>3.4</td>
<td>0.83</td>
<td>2.84</td>
<td>-20.0 to 18.7</td>
</tr>
<tr>
<td>5 n d1</td>
<td>3295</td>
<td>0.92</td>
<td>-1.1</td>
<td>0.96</td>
<td>-1.40</td>
<td>-15.1 to 16.6</td>
</tr>
<tr>
<td>6 n d1 mf</td>
<td>2030</td>
<td>0.92</td>
<td>-1.6</td>
<td>0.98</td>
<td>-1.12</td>
<td>-18.1 to 13.9</td>
</tr>
<tr>
<td>7 n d3</td>
<td>3295</td>
<td>0.92</td>
<td>2.0</td>
<td>0.93</td>
<td>1.30</td>
<td>-11.2 to 19.0</td>
</tr>
<tr>
<td>8 n d3 mf</td>
<td>2061</td>
<td>0.92</td>
<td>1.5</td>
<td>0.94</td>
<td>1.70</td>
<td>-15.1 to 16.4</td>
</tr>
<tr>
<td>9 w d1</td>
<td>2645</td>
<td>0.93</td>
<td>2.0</td>
<td>1.0</td>
<td>2.01</td>
<td>-27.0 to 14.0</td>
</tr>
<tr>
<td>10 w d1 mf</td>
<td>2364</td>
<td>0.93</td>
<td>1.6</td>
<td>0.99</td>
<td>1.75</td>
<td>-27.2 to 13.0</td>
</tr>
<tr>
<td>11 w d3</td>
<td>2645</td>
<td>0.94</td>
<td>5.1</td>
<td>0.96</td>
<td>5.26</td>
<td>-22.8 to 16.6</td>
</tr>
<tr>
<td>12 w d3 mf</td>
<td>2386</td>
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larger than the range for individual segments. At $Z_{\text{obs}} = -20 \text{ dBZ}$ the spread between the two dash–dotted lines is a factor of 50, whereas the mean value of the 90% variation of segments near that observed reflectivity value is a factor of 12.5. At $Z_{\text{obs}} = +10 \text{ dBZ}$, the spread of 1-Hz values (the dash–dotted lines) is a factor of 6 while the variation within segments is a factor of 2.5. As these numbers indicate, the variability of $Z_{\text{calc}}$ for given $Z_{\text{obs}}$ is considerably larger for the 1-Hz values than within specific data segments. From this it can be inferred that 2D-C sampling and data-processing limitations contribute a smaller part of the scatter of points in Figs. 7 and 8 and that the majority of that scatter is due to variations in ice crystal types or densities.

An additional factor that influences the comparison of observed and calculated reflectivities is revealed by examination of the coefficient of variation (CV: standard deviation divided by the mean) of the reflectivity (mm$^2$ m$^{-3}$). The CV values for the observed data have a negatively skewed distribution with a peak near CV = 0.25. The distribution of the calculated values is closely similar in shape but is shifted toward higher values by about 0.15. Whereas 30 flight segments have CV$_{\text{obs}} < 0.15$, none of the CV$_{\text{calc}}$ values fall below that limit. This offset in CV represents a level of noise in the calculated reflectivity that is a likely result of the small sampling volume of the 2D-C probe.

The relationship between observed and calculated reflectivities, and the functions relating IWC and PR to the observed reflectivity, are not seriously affected by the scatter of points, since the range of variables is large and the overall trends are dominated by variations within that range. The best demonstration of this is that the $Z_{\text{calc}}$-versus-$Z_{\text{obs}}$ function as well as the IWC- and PR-versus-$Z_{\text{obs}}$ functions (as will be discussed later) are nearly identical when determined using the 1-Hz data or the flight-segment means. Tables 2 and 3 contain the relevant constants of the equations under the “1-Hz” and “segments” headings. These results show that averaging over periods of 16–240 s does not change the key results.

To examine further the sources of variability, the range of variation of observed and measured reflectivities was calculated as a function of the length of flight segments. For flight segments of $<20$, 20–40, and 40–60-s duration (approximately 2, 2–4, and 4–6 km) the mean values of

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**Fig. 8.** Scatterplot of the measured reflectivity $Z_{\text{obs}}$ vs the calculated reflectivity (for the d1 density assumption) from the 50-percentile values for each flight segment. The vertical and horizontal bars on selected points extend to the 5- and 95-percentile values for that flight segment. The solid line shows the best-fit linear relationship to the points and the dash–dotted lines bracket 90% of the 1-Hz points as in Fig. 7.

**Fig. 9.** Measured reflectivity vs reflectivity difference (calculated minus measured reflectivity). The solid line with squares is for the calculated reflectivity from the d1 density assumption, and the solid line with circles is for the calculated reflectivity from the d3 density assumption. Numbers indicate how many seconds of data are represented by selected points.
the difference between the 5- and 95-percentile values of $Z_{\text{calc}}$ are 6.1, 13.1, and 12.2 mm$^6$ m$^{-3}$, respectively. The same sequence for $Z_{\text{obs}}$ is 1.4, 4.7, and 4.0 mm$^6$ m$^{-3}$, This comparison reveals that, as the sampling period increases, variability of cloud composition more than offsets the gain of averaging over larger sample volumes. That finding holds for both the 2D probe and for the radar.

### c. Sensitivity of the results to data characteristics

As a possible indication of systematic differences in the relationship between $Z_{\text{calc}}$ and $Z_{\text{obs}}$ depending on crystal type, the data were stratified by the temperature measured at flight level. The use of temperature as a surrogate for direct detection of crystal type is based on the well-established pattern of change in crystal growth habit with temperature. No differences could be ascertained, even though the observations covered a wide range of temperatures and variations in crystal habit can be expected to cause deviations from the formula used for $Z_{\text{calc}}$. The same conclusion of no systematic pattern could be drawn from a plot of the difference between observed and calculated reflectivities versus temperature. This lack of temperature dependence in the data may be due to a combination of two facts: rimed crystals and other irregular shapes dominate the data, justifying the assumptions made in obtaining $Z_{\text{calc}}$, and crystals encountered near the flight level often originated at other altitudes, thereby making correlations with temperature at flight level less likely to emerge.

The overall dataset yielded little regarding the role of crystal habit, but some cases were found in which pristine crystal types could be seen in the images, although even in these cases a mixture of identifiable crystals and irregular ones were present. Dendrites were the most readily identified. For a selection of these cases it was found that the calculated reflectivity deviated from the best-fit prediction by from $-1$ to $+5$ dBZ, values that are well within the scatter of points for the overall dataset.

### d. Mass–reflectivity relationship

The significant correlation found between the reflectivity measured with the W-band radar and the reflectivity calculated from particle-imaging data for ice crystals provides a reasonable degree of justification for using these data to examine the relationship between ice mass and reflectivity. IWC is a difficult parameter to measure, and having reliable estimates of it from remote sensing data can be of value for descriptions of cloud processes, even with the uncertainties involved. High resolution and large volume coverage of the radar data are important strengths in this regard.

Because reflectivity depends on particle size with an exponent roughly 2 times as large as for mass, errors in particle sizing should have a lesser impact on the calculations of mass than on the reflectivity, and so the mass–reflectivity relationship can be expected to be as strong or stronger than that of measured versus calculated reflectivity.

The relationship between IWC and $Z_{\text{obs}}$ is shown in Fig. 10. Data included here are by flight segments as in Fig. 8. Symbols indicate the 50-percentile values; vertical and horizontal bars extend to the 5- and 95-percentile values. For clarity, only selected points have the range bars shown. The solid line in Fig. 10 is the best-fit linear regression (correlation coefficient $r = 0.92$) between log(IWC) and $Z_{\text{obs}}$ (dBZ), using the 1-Hz values. The dash–dotted lines bracket 90% of the points as determined by evaluating the 5% and 95% limits in intervals of 5 dBZ. The dotted line is the best fit to the medians for the flight segments.

From the best-fit linear regression equations the mass–reflectivity relation is expressed as $\text{IWC} = a(Z_{\text{obs}}/Z_o)^b$, where IWC has units of grams per cubic meter, $Z_{\text{obs}}$ is in millimeters to the sixth power per meter cubed, $Z_o = 1 \text{ mm}^6 \text{ m}^{-3}$, and $a$ and $b$ are constants whose values are summarized in Table 3 for the various data stratifications. For simplicity, common usage omits the normalization to $Z_o$; that practice is followed in some later parts of this paper.
As expected, the higher particle density assumption $d_3$ leads to higher values of IWC. Because the slope values are very similar, the $1.5$ ratio of the $a$ values reflects the approximate magnitudes of the change for all values of $Z_{\text{obs}}$. Restricting the data to points in the mf subset (80% of IWC accumulated by crystals of $<1$ mm) leads to only small alterations in the coefficients.

e. Precipitation rate–reflectivity relationship

Figure 11 shows the scatterplot of precipitation rate versus radar reflectivity using the 50-percentile value for each time segment and with horizontal and vertical bars extending to the 5- and 95-percentile values for that flight segment. The solid line shows the best-fit linear relationship to the points. The best-fit line to the medians for the flight segments is indistinguishable from this line. The 5- and 95-percentile limits for the overall dataset are included as dash–dotted lines.

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4. Discussion

Of the two instruments employed in this study, the radar and the 2D-C probe, the latter has the larger number of potential error sources. Limitations arise from the operating principle and design of the probe and from the manner of data extraction. The impacts of these limitations have been examined in several ways. First, as described in section 2a, the comparison with data derived from a 2D-P probe provided evidence that the larger sample volume of the 2D-P probe did not increase the derived reflectivity and could even lead to lower values. The same conclusion can be expected to hold for lower moments of the size distributions. Second, stratification by the relative contributions of crystals of $<1$ mm led to correlations between observed and calculated reflectivities that were nearly identical to those of the full dataset (cf. Table 2). Third, the observed and computed reflectivities based on 2D-C data have a nearly 1-to-1 agreement over five orders of magnitude in reflectivities; this is post hoc evidence but nonetheless is strongly supportive. It must be emphasized, however, that these findings are specific to the dataset used in this study and cannot be generalized without further evaluations.

The general degree of agreement seen between calculated and observed reflectivities (Fig. 7 and Table 2) is better than expected, but it is accompanied by a considerable degree of scatter. In light of the approximately $\pm 1$ dBZ stability of the WCR measurements, the spread of points can be interpreted, as discussed earlier, as being due to a combination of errors in the 2D-C data and to actual and undiagnosed deviations from the assumptions used in deriving reflectivity from the image data.

Longer averaging times and correspondingly larger sampled volumes do not improve significantly the
agreement between measured and calculated reflectivity. This is readily seen in that the correlation coefficient for the 50-percentile points of flight segments is 0.96, only slightly larger than the value of 0.94 for the 1-Hz data points (Table 2), and that the 90% ranges of the 1-Hz points seem to also hold only a slightly larger proportion of the mean values for flight segments (dash–dotted lines in Fig. 8). Thus, the conclusion is warranted that the scatter is not due to instrument noise but to other underlying causes.

As seen in Fig. 7, up to ±10-dBZ variation is found in calculated reflectivities near the lowest values of the observed reflectivity. In converse, for a given predicted value there is a ±10-dBZ variation in the observed value. At the upper end of the range of values the scatter is about ±3 dBZ (for a 90% likelihood). Finding decreasing uncertainty for higher values is counterintuitive from the point of view of possible errors with the 2D-C probe arising from particle shattering, undersampling of large particles, or other artifacts. An indication that there is no simple explanation for the trend is given by the fact that the difference \( Z_{\text{obs}} - Z_{\text{calc}} \) has no correlation with ice particle concentration \( r \approx 0.1 \) and only a very weak one with ice water content \( r \approx -0.3 \). The main remaining factors that could be important are variations in ice particle shape and density. The reduced scatter at higher reflectivities is perhaps an indication that crystals evolve toward small graupel or complex shapes independent of their earlier form.

The correlations between measured reflectivity and calculated ice water content and between reflectivity and precipitation rate are slightly lower \( r = 0.92 \) and \( r = 0.93 \) than for measured versus calculated reflectivity \( r = 0.96 \). This goes against the expectation that errors in sizing will be less important for lower moments of the size distribution. So, this evidence too points to the major uncertainties arising from shape and density assumptions.

The \( IWC-Z \) relationship from this work is very close to that reported by LI00 for 94 GHz. Those authors calculated both the mass and reflectivity from ice particle size distributions they observed with a 2D-C probe in various field programs. Their results are compared with ours in Fig. 12, where the results from this work are plotted as a function of the observed reflectivity. The \( IWC \) values predicted from our equations are somewhat higher than those of LI00 for the range covered by both datasets (<7 dBZ). Two lines are shown from LI00: one for what they consider to be a consensus result from a number of experiments, and a second line at lower \( IWC \) for a different assumption for particle density. Detailed explorations of the effects of different assumptions about particle density and temperature in that work showed that variations of roughly 30% in \( IWC \) can result from those factors. That variation is small, however, in comparison with the factor-of-2-or-more scatter in the estimated values of \( IWC \), and even that factor-of-2 estimate is low relative to the scatter of \( Z_{\text{obs}} \) versus \( Z_{\text{calc}} \) found in the physically more tightly coupled measurements presented here. Protat et al. (2007) found that, with minor adjustments, the relationships of LI00 held well for their large dataset.

Matrosov [2007 (M07)] calculated the relationship between precipitation rate and reflectivity at 94 GHz assuming exponential size distributions. These calculations refer to dendrites and aggregates that are of low density relative to the crystals in our data. M07 report values of \( c \) that vary with density and with fall velocity in the range \( c = 6-12 \), and the exponent is \( d = 0.8 \) for all cases. The comparison of our results with the estimates given by M07 is shown in the right-hand panel of Fig. 12. The predicted precipitation rate from our data, which is for denser particles and higher fall velocities for given sizes, is larger than the M07 values. The two sets of curves converge at high values of \( Z \).

5. Summary and conclusions

Radar reflectivity at 94 GHz measured in the close vicinity of the Wyoming King Air research aircraft and the reflectivity calculated from in situ measurements of ice particle size distributions were found to have good quantitative correspondence over the range from −18 to +16 dBZ of observed values (for 90% of 5940 1-s averages). The data originated from flights in basically stratiform mountain clouds during two wintertime projects and covered the temperature range from −37° to −10°C. Except for a few traces of liquid droplets, the clouds contained only ice crystals. The maximum sizes of the crystals were a few millimeters, and they were mostly of irregular, graupel-like shapes.

The agreement we found between radar and in situ data \( r > 0.92 \) is consistent with results reported by other authors (e.g., Wang et al. 2005; Protat et al. 2007). The two types of measurements are more closely coordinated in our work than in the previous ones, because both the radar and the in situ measurements were taken from the same aircraft. Mismatch of location is thus eliminated, and the sampling volume disparity is significantly reduced. Crystal size distributions were obtained with a 2D-C imaging probe. In the absence of direct measurements of ice water content, particle sizes combined with a size-density function were used to calculate the \( IWC \). Particle density and fall-velocity values that were used in the calculations of reflectivity, \( IWC \), and precipitation rate were based on values reported in the literature for these types of crystals. Because those values are based on scant data, and to bracket the
potential range of particle densities, two different functions of density versus size were used, and results are reported for both. Overall, considering the various data groupings shown in Table 2, the lower density assumption (d1) yielded closer correspondence to observed reflectivities. The evaluation of IWC, PR, and $Z_{\text{calc}}$ from particle data has several potential error sources. The 2D-C particle imaging probe has a small sampling volume and limited resolution, particle density and fall velocity were assumed to be described by the same function for all clouds, and the calculation of reflectivity involved uncertainties in the value of the refractive index and in the application of Mie equations. The precise effects of these potential errors are not well known.

Power-law equations that relate IWC and PR to the observed reflectivity were derived, and the coefficients of the equations are reported in Table 3. Correlations between these parameters and $Z_{\text{obs}}$ is as strong as that between $Z_{\text{obs}}$ and $Z_{\text{calc}}$. The equations, in their simplified forms, for the d1 density function are $IWC = 0.10Z^{0.51}$ and $PR = 0.39Z^{0.58}$. Here, $Z$ is in millimeters to the sixth power per meter cubed, IWC is in grams per meter cubed, and PR is in millimeters per hour. These equations are valid for the range from $-25$ to $+15$ dBZ. The equation for IWC predicts values that agree with the results of LI00 to within a factor of 2 despite different methods being used in the two works. The equation for PR has no good comparison in the literature; the closest are the equations given in M07, but those are for dendrite and aggregate crystal types that have lower densities than our samples. The many more equations available for other radar wavelengths are not directly comparable to the ones given here for 94 GHz.

The strong correlation between observed and calculated reflectivity is accompanied by considerable scatter for both 1 s averaging times and for longer averaging times. Because the calculations assumed the same size-versus-density function for all cases, variations from those assumptions are one evident source of the scatter. Increases in the sampling period and inherently greater variability of cloud composition were found to more than offset the gain of averaging over larger sample volumes. Stratification by temperature and thus by expected growth habit did not reduce the scatter significantly, in part because the crystals grew at different altitudes from the sampling level. In these data, increasing the averaging times did not lead to reduced scatter, and so the larger sample volumes of ground-based or satellite-based radars cannot be expected to yield greater accuracies in predicted cloud parameters. Because of the unknown
magnitude of the contribution of errors in probe data and in the application of simplified derivations of reflectivity, the observed spread of values can be viewed as an upper limit to the imprecision that can be expected in predictions of ice water content and precipitation rate from radar reflectivity alone. In rough terms, this spread is a factor of 10 for 90% confidence—less at high values and somewhat worse at low ones. Because of the evidence found here that density and shape variations of crystals are responsible for much of the scatter, it is not unreasonable to consider the factor-of-10 uncertainty in radar-derived cloud parameters to be a realistic limitation of these indirect assessments of cloud composition until improvements are identified. Such improvements can come from the addition of other radar parameters (e.g., velocity or polarization), from coincident measurements by other instruments, or from constraints within models.

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APPENDIX

Data Processing for Particle Images from 2D-C Probe

a. Artifact rejection

Artifact rejection is based on timing, image shape, and edge overlap. Three basic measures are used for each image:

1) $ix =$ maximum number of shadowed bits along the flight direction between timing words,
2) $iy =$ maximum number of shadowed bits across the detector array, and
3) $ia =$ total number of shadowed bits for the image.

There are nine rejection criteria:

1) The first particle after a buffer overload is rejected because it is not complete and the timer word is bad.
2) If the distance between particles is too small based on what would be expected from the concentration, the particle will be rejected.
3) The particle is rejected if there are more than two complete arraywide gaps in the image.
4) Zero-area images, $ia = 0$ (the counter is triggered but no diodes are shadowed), are rejected.
5) The particle is rejected if $ia < 0.2 \times ix \times iy$.
6) Slow-moving particles will appear to have their edges parallel to the edges of the image strip. If more than one-half of the total length is parallel to the edges, the particle is rejected unless the $ix < 1.5 \times iy$.
7) If $ix > 1$ cm, it is rejected as a streaker.
8) If $ix < 600 \mu m$ and $iy < ix/6$, it is rejected as a streaker.
9) If $ix > 600 \mu m$ and $iy < ix/3$, the particle is rejected as a streaker, except if it overlaps an edge of the array over more than $0.5 \times ix$.

b. Size determination

Determination of the “particle size” from irregularly shaped shadow images has been approached many different ways in the literature, ranging from use of one of the maximum image dimensions to elaborate pattern-recognition routines. Our approach is between the extremes and probably closer to the simple end. It is based on fitting a prolate spheroid to the shadow image and calculating the diameter of a sphere of the same volume $D_{ps}$.

Using a least squares routine, the length and orientation of the maximum dimension of the image are determined and are taken as the polar axis of the prolate spheroid. An axis perpendicular to this is taken as the equatorial diameter. By using standard formulas for the volumes of the prolate spheroid and the sphere, $D_{ps}$ is readily obtained. For images that are not cut off by the limited width of the detector array (0.08 cm), this routine has no major drawbacks. For partial images, the routine affords some compensation for the missing portions but the effectiveness of this approach cannot be determined.

The value of $D_{ps}$ for each particle is taken as the basis for sorting particles into 20 bins of progressively greater widths. The first bin spans 0–50 $\mu m$, and the last spans 6000–7000 $\mu m$. For this work, counts accumulated over intervals of 1 s were used. Integrated quantities are derived from these size distributions by using the bin center as the nominal size.

REFERENCES


