ERRATUM TO “MINIMUM NUMBER OF DISTINCT EIGENVALUES OF GRAPHS”

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In this note, we wish to add an additional hypothesis to Theorem 4.4, which in turn impacts the resulting original conclusions of Corollaries 4.5 and 4.6 as presented in the paper [Minimum Number of Distinct Eigenvalues of Graphs, The Electronic Journal of Linear Algebra, Volume 26, pp. 673-691, 2013].

Consider the following necessary notation. Suppose $\alpha \subseteq \{1, 2, \ldots, m\}$ and $\beta \subseteq \{1, 2, \ldots, n\}$. For a matrix $A \in M_{m,n}$, $A[\alpha, \beta]$ denotes the submatrix of $A$ lying in rows indexed by $\alpha$ and columns indexed by $\beta$. For any vertex $v$ of a graph $G$, the neighborhood set of $v$, denoted by $N(v)$, is the set of all vertices in $G$ adjacent to $v$.

**Theorem 4.4** Let $G$ be a connected graph on $n$ vertices with $q(G) = 2$. Then, for any independent set of vertices $\{v_1, v_2, \ldots, v_k\}$, that satisfies for each $i = 1, 2, \ldots, k$ there exists a $j \neq i$ for which $N(v_i) \cap N(v_j) \neq \emptyset$, we have

$$\left| \bigcup_{i \neq j} (N(v_i) \cap N(v_j)) \right| \geq k.$$

In the original version of this theorem (see Theorem 4.4 in the aforementioned paper) we inadvertently omitted the possibility that for some $i$, $N(v_i) \cap N(v_j)$ is empty for each $j \neq i$. Assuming that this possibility does not arise, the original proof goes through as it appears. As noted in our work, the next two results are immediate consequences of Theorem 4.4; however, we need to account for the additional case that was previously omitted. We include the revised statements of both results here for clarity and completeness.

**Corollary 4.5** Let $G$ be a connected graph on $n \geq 3$ vertices with $q(G) = 2$. Then, any two non-adjacent vertices must have at least two common neighbors or none at all.

**Corollary 4.6** Suppose $q(G) = 2$, for a connected graph $G$ on $n \geq 3$ vertices. If a vertex $v_1$ has degree exactly two with adjacent vertices $v_2$ and $v_3$, then every vertex $v$, different from $v_2$ and $v_3$, has exactly the same neighbors as $v_1$ or has no common neighbors with $v_1$. 