CONVERGENCE ON GAUSS-SEIDEL ITERATIVE METHODS FOR LINEAR SYSTEMS WITH GENERAL $H$–MATRICES

CHENG-YI ZHANG†, DAN YE‡, CONG-LEI ZHONG§, AND SHUANGHUA LUO¶

Abstract. It is well known that as a famous type of iterative methods in numerical linear algebra, Gauss-Seidel iterative methods are convergent for linear systems with strictly or irreducibly diagonally dominant matrices, invertible $H$–matrices (generalized strictly diagonally dominant matrices) and Hermitian positive definite matrices. But, the same is not necessarily true for linear systems with nonstrictly diagonally dominant matrices and general $H$–matrices. This paper firstly proposes some necessary and sufficient conditions for convergence on Gauss-Seidel iterative methods to establish several new theoretical results on linear systems with nonstrictly diagonally dominant matrices and general $H$–matrices. Then, the convergence results on preconditioned Gauss-Seidel (PGS) iterative methods for general $H$–matrices are presented. Finally, some numerical examples are given to demonstrate the results obtained in this paper.

Key words. Gauss-Seidel iterative methods, Convergence, Nonstrictly diagonally dominant matrices, General $H$–matrices.

AMS subject classifications. 15A15, 15F10.