SOLVING THE REAL EIGENVALUES OF HERMITIAN QUADRATIC EIGENVALUE PROBLEMS VIA BISECTION

HAO LI† AND YUNFENG CAI†

Abstract. This paper considers solving the real eigenvalues of the Quadratic Eigenvalue Problem (QEP) \( Q(\lambda)x \equiv (\lambda^2 M + \lambda C + K)x = 0 \) in a given interval \((a, b)\), where the coefficient matrices \(M\), \(C\), \(K\) are Hermitian and \(M\) is nonsingular. First, an inertia theorem for the QEP is proven, which characterizes the difference of inertia index between Hermitian matrices \(Q(a)\) and \(Q(b)\). Several useful corollaries are then obtained, where it is shown that the number of real eigenvalues of QEP \( Q(\lambda)x = 0 \) in the interval \((a, b)\) is no less than the absolute value of the difference of the negative inertia index between \(Q(a)\) and \(Q(b)\); furthermore, when all real eigenvalues in \((a, b)\) are semi-simple with the same sign characteristic, the inequality becomes an equality. Based on the established theory, the bisection method (with preprocessing) can be used to compute the real eigenvalues of the QEP by computing the inertia indices. Applications to the calculation of the equi-energy lines with \(k.p\) model, and also a nonoverdamped mass-spring system are presented in the numerical tests.

Key words. Quadratic eigenvalue problem, Index of inertia, Real eigenvalue, Bisection.

AMS subject classifications. 65F15, 65Z05.

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†LMAM & School of Mathematical Sciences, Peking University, Beijing, 100871, China (lihao.mathematics@gmail.com, yfcai@math.pku.edu.cn). This research was supported by NSFC under Grant No. 11301013.