THE LAPLACIAN QUADRATIC FORM AND EDGE CONNECTIVITY OF A GRAPH

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Abstract. Let $G$ be a simple connected graph with associated positive semidefinite integral quadratic form $Q(x) = \sum (x(i) - x(j))^2$, where the sum is taken over all edges $ij$ of $G$. It is showed that the minimum positive value of $Q(x)$ for $x \in \mathbb{Z}^n$ equals the edge connectivity of $G$. By restricting $Q(x)$ to $x \in \mathbb{Z}^{n-1} \times \{0\}$, the quadratic form becomes positive definite. It is also showed that the number of minimal disconnecting sets of edges of $G$ equals twice the number of vectors $x \in \mathbb{Z}^{n-1} \times \{0\}$ for which the form $Q$ attains its minimum positive value.

Key words. Graph, Laplacian matrix, Edge connectivity, Integral quadratic form.

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