NOTE ON THE JORDAN FORM OF AN IRREDUCIBLE EVENTUALLY NONNEGATIVE MATRIX

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Abstract. A square complex matrix $A$ is eventually nonnegative if there exists a positive integer $k_0$ such that for all $k \geq k_0$, $A^k \geq 0$; $A$ is strongly eventually nonnegative if it is eventually nonnegative and has an irreducible nonnegative power. It is proved that a collection of elementary Jordan blocks is a Frobenius Jordan multiset with cyclic index $r$ if and only if it is the multiset of elementary Jordan blocks of a strongly eventually nonnegative matrix with cyclic index $r$. A positive answer to an open question and a counterexample to a conjecture raised by Zaslavsky and Tam are given. It is also shown that for a square complex matrix $A$ with index at most one, $A$ is irreducible and eventually nonnegative if and only if $A$ is strongly eventually nonnegative.

Key words. Irreducible eventually nonnegative, Strongly eventually nonnegative, Eventually reducible, Eventually $r$-cyclic, Cyclic index, Frobenius collection, Frobenius Jordan multiset, Jordan multiset, Jordan form.

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