ON THE NOTION OF SCALAR PRODUCT FOR FINITE-DIMENSIONAL DIFFELOGICAL VECTOR SPACES

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Abstract. It is known that the only finite-dimensional diffeological vector space that admits a diffeologically smooth scalar product is the standard space of appropriate dimension. In this note, a way to dispense with this issue is considered, by introducing a notion of pseudo-metric, which, said informally, is the least-degenerate symmetric bilinear form on a given space. This notion is applied to make some observations on subspaces which split off as smooth direct summands (providing examples which illustrate that not all subspaces do), and then to show that the diffeological dual of a finite-dimensional diffeological vector space always has the standard diffeology and in particular, any pseudo-metric on the initial space induces, in the obvious way, a smooth scalar product on the dual.

Key words. Diffeological space, Diffeological vector space, Diffeologically smooth bilinear forms, Diffeological pseudo-metric.

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