THE SUM OF THE FIRST TWO LARGEST SIGNLESS LAPLACIAN EIGENVALUES OF TREES AND UNICYCLIC GRAPHS∗

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Abstract. Let $G$ be a graph on $n$ vertices with $e(G)$ edges. The sum of eigenvalues of graphs has been receiving a lot of attention these years. Let $S_2(G)$ be the sum of the first two largest signless Laplacian eigenvalues of $G$, and define $f(G) = e(G) + 3 - S_2(G)$. Oliveira et al. (2015) conjectured that $f(G) \geq f(U_n)$ with equality if and only if $G \cong U_n$, where $U_n$ is the $n$-vertex unicyclic graph obtained by attaching $n - 3$ pendent vertices to a vertex of a triangle. In this paper, it is proved that $S_2(G) < e(G) + 3 - 2n$ when $G$ is a tree, or a unicyclic graph whose unique cycle is not a triangle. As a consequence, it is deduced that the conjecture proposed by Oliveira et al. is true for trees and unicyclic graphs whose unique cycle is not a triangle.

Key words. The sum of eigenvalues, Signless Laplacian eigenvalues, Laplacian eigenvalues, Trees, Unicyclic graphs.

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