REFINED INERTIA OF MATRIX PATTERNS

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Abstract. This paper explores how the combinatorial arrangement of prescribed zeros in a matrix affects the possible eigenvalues that the matrix can obtain. It demonstrates that there are inertially arbitrary patterns having a digraph with no 2-cycle, unlike what happens for nonzero patterns. A class of patterns is developed that are refined inertially arbitrary but not spectrally arbitrary, making use of the property of a properly signed nest. The paper includes a characterization of the inertially arbitrary and refined inertially arbitrary patterns of order three, as well as the patterns of order four with the least number of nonzero entries.

Key words. Spectrally arbitrary pattern, Refined inertia, Inertially arbitrary pattern.

AMS subject classifications. 15A18, 15A29, 15B35.

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