LIGHTS OUT! ON CARTESIAN PRODUCTS*

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Abstract. The game LIGHTS OUT! is played on a $5 \times 5$ square grid of buttons; each button may be on or off. Pressing a button changes the on/off state of the light of the button pressed and of all its vertical and horizontal neighbors. Given an initial configuration of buttons that are on, the object of the game is to turn all the lights out. The game can be generalized to arbitrary graphs. In this paper, Cartesian Product graphs (that is, graphs of the form $G \square H$, where $G$ and $H$ are arbitrary finite, simple graphs) are investigated. In particular, conditions for which $G \square H$ is universally solvable (every initial configuration of lights can be turned out by a finite sequence of button presses), using both closed neighborhood switching and open neighborhood switching, are provided.

Key words. Matrix, Determinant, Graph, Lights Out, Fibonacci polynomials.

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