NORM INEQUALITIES RELATED TO CLARKSON INEQUALITIES∗

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Abstract. Let $A$ and $B$ be $n \times n$ matrices. It is shown that if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both $A + B$, $A - B$ are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \leq 2^{p-1} \left(\|A\|_p^p + \|B\|_p^p\right) - \left(2^{p/2} - 2\right) \left(\|A\|_p - \|B\|_p\right)_p^p,$$

and if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both $A$, $B$ are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \geq 2 \left(\|A\|_p^p + \|B\|_p^p\right) + \left(2^{1-p/2} - 2^{2-p}\right) \left(\|A + B\|_p - \|A - B\|_p\right)_p^p.$$

These inequalities are reversed if $p = 2$, $1 \leq p < \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both $A + B$, $A - B$ are positive semidefinite, and if $p = 2$, $1 \leq p \leq \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both $A$, $B$ are positive semidefinite, respectively. Commutative (or $L_p$) versions of these inequalities are also considered.

Key words. Clarkson inequalities, Hanner’s inequality, Schatten $p$-norm, $L_p$ function, Singular value.

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