RANK FUNCTION AND OUTER INVERSES*

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Abstract. For the class of matrices over a field, the notion of ‘rank of a matrix’ as defined by ‘the dimension of subspace generated by columns of that matrix’ is folklore and cannot be generalized to the class of matrices over an arbitrary commutative ring. The ‘determinantal rank’ defined by the size of largest submatrix having nonzero determinant, which is same as the column rank of given matrix when the commutative ring under consideration is a field, was considered to be the best alternative for the ‘rank’ in the class of matrices over a commutative ring. Even this determinantal rank and the McCoy rank are not so efficient in describing several characteristics of matrices like in the case of discussing solvability of linear system. In the present article, the ‘rank–function’ associated with the matrix as defined in [Solvability of linear equations and rank–function, K. Manjunatha Prasad, http://dx.doi.org/10.1080/00927879708825854] is discussed and the same is used to provide a necessary and sufficient condition for the existence of an outer inverse with specific column space and row space. Also, a rank condition is presented for the existence of Drazin inverse, as a special case of an outer inverse, and an iterative procedure to verify the same in terms of sum of principal minors of the given square matrix over a commutative ring is discussed.

Key words. rank–function, generalized inverse, outer inverse, Drazin inverse, matrix over a commutative ring

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