SPECTRAL BOUNDS FOR THE CONNECTIVITY OF REGULAR GRAPHS WITH GIVEN ORDER

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Abstract. The second-largest eigenvalue and second-smallest Laplacian eigenvalue of a graph are measures of its connectivity. These eigenvalues can be used to analyze the robustness, resilience, and synchronizability of networks, and are related to connectivity attributes such as the vertex- and edge-connectivity, isoperimetric number, and characteristic path length. In this paper, two upper bounds are presented for the second-largest eigenvalues of regular graphs and multigraphs of a given order which guarantee a desired vertex- or edge-connectivity. The given bounds are in terms of the order and degree of the graphs, and hold with equality for infinite families of graphs. These results answer a question of Mohar.

Key words. Second-largest eigenvalue, Vertex-connectivity, Edge-connectivity, Regular multigraph, Algebraic connectivity.

AMS subject classifications. 05C50, 05C40.

*Received by the editors on November 30, 2017. Accepted for publication on July 18, 2018. Handling Editor: Sebastian M. Cioabă. Corresponding Author: Suil O.
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