Abstract. For a proper cone $K$ in a finite dimensional real Hilbert space $V$, a linear map $L$ is said to be $K$-semipositive if there exists $d \in K^\circ$, the interior of $K$, such that $L(d) \in K^\circ$. The aim of this manuscript is to characterize $K$-semipositivity of linear maps relative to a proper cone. Among several results obtained, $K$-semipositivity is characterized in terms of products of the form $YX^{-1}$ for $K$-positive linear maps ($L(K \setminus \{0\}) \subseteq K^\circ$) with $X$ invertible, semipositivity of matrices relative to the $n$-dimensional Lorentz cone $L_n^+$ is characterized, semipositivity of the following three linear maps relative to the cone $S_n^+$: $X \mapsto AXB$ (denoted by $M_{A,B}$), $X \mapsto AXB + B^tXA^t$ (denoted by $L_{A,B}$), where $A, B \in M_n(\mathbb{R})$, and $X \mapsto X - AXA^t$ (denoted by $S_A$, known as the Stein transformation) is characterized. It is also proved that $M_{A,B}$ is semipositive if and only if $B = \alpha A^t$ for some $\alpha > 0$, the map $L_{A,B}$ is semipositive if and only if $A(B^t)^{-1}$ is positive stable. A particular case of the new result generalizes Lyapunov’s theorem. Decompositions of the above maps (when they are semipositive) in the form $L_1L_2^{-1}$, where $L_1$ and $L_2$ are both positive and invertible (assuming $A$ is invertible in the case of $S_A$) are presented. Moreover, a question on invariance of the semipositive cone $K_A$ of a matrix under $A$ is partially answered.

Key words. Positivity and semipositivity of linear maps, Proper cones, Positive definite matrices, Positive stable matrices, Semidefinite linear complementarity problems, Lyapunov and Stein transformations, Semipositive cone.

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