DIAGONAL SUMS OF DOUBLY SUBSTOCHASTIC MATRICES

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Abstract. Let \( \Omega_n \) denote the convex polytope of all \( n \times n \) doubly stochastic matrices, and \( \omega_n \) denote the convex polytope of all \( n \times n \) doubly substochastic matrices. For a matrix \( A \in \omega_n \), define the sub-defect of \( A \) to be the smallest integer \( k \) such that there exists an \( (n + k) \times (n + k) \) doubly stochastic matrix containing \( A \) as a submatrix. Let \( \omega_{n,k} \) denote the subset of \( \omega_n \) which contains all doubly substochastic matrices with sub-defect \( k \). For \( \pi \) a permutation of symmetric group of degree \( n \), the sequence of elements \( a_{1\pi(1)}, a_{2\pi(2)}, \ldots, a_{n\pi(n)} \) is called the diagonal of \( A \) corresponding to \( \pi \). Let \( h(A) \) and \( l(A) \) denote the maximum and minimum diagonal sums of \( A \in \omega_{n,k} \), respectively. In this paper, existing results of \( h \) and \( l \) functions are extended from \( \Omega_n \) to \( \omega_{n,k} \). In addition, an analogue of Sylvesters law of the \( h \) function on \( \omega_{n,k} \) is proved.

Key words. Doubly substochastic matrices, Sub-defect, Maximum diagonal sum.

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