ON THE MAXIMAL NUMERICAL RANGE OF SOME MATRICES∗

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Abstract. The maximal numerical range $W_0(A)$ of a matrix $A$ is the (regular) numerical range $W(B)$ of its compression $B$ onto the eigenspace $\mathcal{L}$ of $A^*A$ corresponding to its maximal eigenvalue. So, always $W_0(A) \subseteq W(A)$. Conditions under which $W_0(A)$ has a non-empty intersection with the boundary of $W(A)$ are established, in particular, when $W_0(A) = W(A)$. The set $W_0(A)$ is also described explicitly for matrices unitarily similar to direct sums of 2-by-2 blocks, and some insight into the behavior of $W_0(A)$ is provided when $\mathcal{L}$ has codimension one.

Key words. Numerical range, Maximal numerical range, Normaloid matrices.

AMS subject classifications. 15A60, 15A57.

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