EFFECTS ON THE DISTANCE LAPLACIAN SPECTRUM OF GRAPHS WITH CLUSTERS BY ADDING EDGES

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Abstract. All graphs considered are simple and undirected. A cluster in a graph is a pair of vertex subsets \((C, S)\), where \(C\) is a maximal set of cardinality \(|C| \geq 2\) of independent vertices sharing the same set \(S\) of \(|S|\) neighbors. Let \(G\) be a connected graph on \(n\) vertices with a cluster \((C, S)\) and \(H\) be a graph of order \(|C|\). Let \(G(H)\) be the connected graph obtained from \(G\) and \(H\) when the edges of \(H\) are added to the edges of \(G\) by identifying the vertices of \(H\) with the vertices in \(C\). It is proved that \(G\) and \(G(H)\) have in common \(n - |C| + 1\) distance Laplacian eigenvalues, and the matrix having these common eigenvalues is given, if \(H\) is the complete graph on \(|C|\) vertices then \(\partial - |C| + 2\) is a distance Laplacian eigenvalue of \(G(H)\) with multiplicity \(|C| - 1\), where \(\partial\) is the transmission in \(G\) of the vertices in \(C\). Furthermore, it is shown that if \(G\) is a graph of diameter at least 3, then the distance Laplacian spectral radii of \(G\) and \(G(H)\) are equal, and if \(G\) is a graph of diameter 2, then conditions for the equality of these spectral radii are established. Finally, the results are extended to graphs with two or more disjoint clusters.

Key words. Cluster, Pendant vertices, Distance Laplacian matrix, Distance Laplacian eigenvalues, Distance spectral radius.

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