CONSISTENCY OF QUATERNION MATRIX EQUATIONS

\[ AX^* - XB = C \] AND \[ X - AX^*B = C^* \]

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Abstract. For a given ordered units triple \( \{q_1, q_2, q_3\} \), the solutions to the quaternion matrix equations \( AX^* - XB = C \) and \( X - AX^*B = C^* \), where \( X^* \) is the conjugate transpose of \( X \), \( X^\eta = -\eta X \eta \) and \( X^{\eta^*} = -\eta X^* \eta \), \( \eta \in \{q_1, q_2, q_3\} \), are discussed. Some new real representations of quaternion matrices are used, which enable one to convert \( \eta \)-conjugate (transpose) matrix equations into some real matrix equations. By using this idea, conditions for the existence and uniqueness of solutions to the above quaternion matrix equations are derived. Also, methods to construct the solutions from some related real matrix equations are presented.

Key words. Quaternion matrix equations, Real representations, \( \eta \)-conjugates, \( \eta \)-conjugate transposes, Ordered units triple.

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