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A NOTE ON THE MATRIX ARITHMETIC-GEOMETRIC MEAN INEQUALITY*

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Abstract. This note proves the following inequality: If $n = 3k$ for some positive integer k, then for any n positive definite matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$, the following inequality holds:

$$\frac{1}{n^3} \Big\| \sum_{j_1, j_2, j_3 = 1}^{n} \mathbf{A}_{j_1} \mathbf{A}_{j_2} \mathbf{A}_{j_3} \Big\| \geq \frac{(n-3)!}{n!} \Big\| \sum_{\substack{j_1, j_2, j_3 = 1, \\ j_1,\ j_2,\ j_3 \text{ all distinct}}}^{n} \mathbf{A}_{j_1} \mathbf{A}_{j_2} \mathbf{A}_{j_3} \Big\|,$$

where $\| \cdot \|$ represents the operator norm. This inequality is a special case of a recent conjecture proposed by Recht and Ré (2012).

Key words. Positive definite matrices, Matrix inequalities.

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