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DIAGONAL SUMS OF DOUBLY SUBSTOCHASTIC MATRICES*

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Abstract. Let Ω_n denote the convex polytope of all $n \times n$ doubly stochastic matrices, and ω_n denote the convex polytope of all $n \times n$ doubly substochastic matrices. For a matrix $A \in \omega_n$, define the sub-defect of A to be the smallest integer k such that there exists an $(n+k) \times (n+k)$ doubly stochastic matrix containing A as a submatrix. Let $\omega_{n,k}$ denote the subset of ω_n which contains all doubly substochastic matrices with sub-defect k. For π a permutation of symmetric group of degree n, the sequence of elements $a_{1\pi(1)}, a_{2\pi(2)}, \ldots, a_{n\pi(n)}$ is called the diagonal of A corresponding to π. Let $h(A)$ and $l(A)$ denote the maximum and minimum diagonal sums of $A \in \omega_{n,k}$, respectively. In this paper, existing results of h and l functions are extended from Ω_n to $\omega_{n,k}$. In addition, an analogue of Sylvesters law of the h function on $\omega_{n,k}$ is proved.

Key words. Doubly substochastic matrices, Sub-defect, Maximum diagonal sum.

AMS subject classifications. 15A51, 15A83.

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