Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 29, pp. 30-45, September 2015

ELA
http://repository.uwyo.edu/ela

A MATRIX HANDLING OF PREDICTIONS UNDER A GENERAL LINEAR RANDOM-EFFECTS MODEL WITH NEW OBSERVATIONS*

YONGGE TIAN†

Abstract. Linear regression models that include random effects are commonly used to analyze longitudinal and correlated data. Assume that a general linear random-effects model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ with $\boldsymbol{\beta} = \mathbf{A}\boldsymbol{\alpha} + \boldsymbol{\gamma}$ is given, and new observations in the future follow the linear model $\mathbf{y}_f = \mathbf{X}_f\boldsymbol{\beta} + \boldsymbol{\varepsilon}_f$. This paper shows how to establish a group of matrix equations and analytical formulas for calculating the best linear unbiased predictor (BLUP) of the vector $\boldsymbol{\phi} = \mathbf{F}\boldsymbol{\alpha} + \mathbf{G}\boldsymbol{\gamma} + \mathbf{H}\boldsymbol{\varepsilon} + \mathbf{H}_f\boldsymbol{\varepsilon}_f$ of all unknown parameters in the two models under a general assumption on the covariance matrix among the random vectors $\boldsymbol{\gamma}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_f$ via solving a constrained quadratic matrix-valued function optimization problem. Many consequences on the BLUPs of $\boldsymbol{\phi}$ and their covariance matrices, as well as additive decomposition equalities of the BLUPs with respect to its components are established under various assumptions.

Key words. Linear random-effects model, Quadratic matrix-valued function, Löwner partial ordering, BLUP, BLUE, Covariance matrix.

AMS subject classifications. 15A09, 62H12, 62J05.

*Received by the editors on February 22, 2015. Accepted for publication on May 28, 2015 Handling Editor: Simo Puntanen.

†China Economics and Management Academy, Central University of Finance and Economics, Beijing 100081, China (yongge.tian@gmail.com). Supported by the National Natural Science Foundation of China (Grant No. 11271384).