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ON HIGMAN'S CONJECTURE*

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Abstract. Let $\mathcal{G}_n$ be the subgroup of $\mathrm{GL}_n(q)$ consisting of the $n \times n$ upper unitriangular matrices over the field $\mathbb{F}_q$ with q elements. Higman [G. Higman. Enumerating p-groups. I. Inequalities. *Proc. London Math. Soc. (3)*, 10:24–30, 1960.] conjectured that the number of conjugacy classes of $\mathcal{G}_n$, denoted by $r(\mathcal{G}_n)$, is a polynomial in q with integer coefficients. This has been verified for $n \leq 13$ by A. Vera-López and J.M. Arregi [A. Vera-López and J.M. Arregi. Conjugacy classes in unitriangular matrices. *Linear Algebra Appl.*, 370:85–124, 2003.]. The main purpose of this paper is to prove that for every n, $r(\mathcal{G}_n)$ can be expressed in terms of $r(\mathcal{G}_i)$, with $i < n$, and $r(\mathcal{T}_n)$, where $\mathcal{T}_n$ is the subset of primitive canonical matrices of $\mathcal{G}_n$. Moreover, the expression of $r(\mathcal{T}_n)$ modulo $(q-1)^{\left[\frac{n+1}{2}\right]+3}$ is determinated and, consequently, it is deduced that $r(\mathcal{T}_n) \mathrm{mod} (q-1)^{\left[\frac{n+1}{2}\right]+3}$ is a polynomial in q with integer coefficients.

Key words. Unitriangular matrices, Higman's conjecture.

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