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NORMALIZED RATIONAL SEMIREGULAR GRAPHS*

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Dedicated to the memory of Prof. David Gregory

Abstract. Let G be a graph and let A and D be the adjacency matrix of G and diagonal matrix of vertex degrees of G respectively. If each vertex degree is positive, then the *normalized adjacency matrix* of G is $\hat{A} = D^{-1/2}AD^{-1/2}$. A classification is given of those graphs for which the all eigenvalues of the normalized adjacency matrix are integral. The problem of determining those graphs G for which $\lambda \in \mathbb{Q}$ for each eigenvalue λ of $\hat{A}(G)$ is considered. These graphs are called *normalized rational*. It will be shown that a semiregular bipartite graph G with vertex degrees r and s is normalized rational if and only if every eigenvalue of A is a rational multiple of $\sqrt{rs}$. This result will be used to classify the values of n for which the semiregular graph (with vertex degrees 2 and $n-1$) obtained from subdividing each edge of K_n is normalized rational. Necessary conditions for the k-uniform complete hypergraph on n vertices to be normalized rational are also given. Finally, conditions for the incidence graphs of Steiner triple and quadruple systems to be normalized rational are given.

Key words. graphs, hypergraphs, semiregular graphs, normalized Laplacian matrix

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