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AN EXPANSION PROPERTY OF BOOLEAN LINEAR MAPS*

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Abstract. Given a finite set K, a Boolean linear map on K is a map f from the set 2^K of all subsets of K into itself with $f(\emptyset) = \emptyset$ such that $f(A \cup B) = f(A) \cup f(B)$ holds for all $A, B \in 2^K$. For fixed subsets X, Y of K, to predict if Y is reachable from X in the dynamical system driven by f, one can assume the existence of nonnegative integers h with $f^h(X) = Y$, find an upper bound α for the minimum of all such assumed integers h, and test if Y really appears in $f^0(X), \ldots, f^\alpha(X)$. In order to get such an upper bound estimate, this paper establishes an expansion property for the Boolean linear map f. Namely, the authors find a lower bound on the size of $f^h(X)$ for any nonnegative integer h. Besides presenting several direct applications of the derived expansion property, this paper collects some related problems on Boolean linear dynamical systems, including problems on primitive multilinear maps and inhomogeneous topological Markov chains.

Key words. Diameter, Digraph, Girth, Phase space, Primitive exponent, Reachability, Wielandt's inequality.

AMS subject classifications. 05C20, 05C50, 68Q45.

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