



## SEMILINEAR PRESERVERS OF THE IMMANANTS IN THE SET OF DOUBLY STOCHASTIC MATRICES\*

M. ANTÓNIA DUFFNER<sup>†</sup> AND ROSÁRIO FERNANDES<sup>‡</sup>

**Abstract.** Let  $S_n$  denote the symmetric group of degree  $n$  and  $M_n$  denote the set of all  $n$ -by- $n$  matrices over the complex field,  $\mathbb{C}$ . Let  $\chi : S_n \rightarrow \mathbb{C}$  be an irreducible character of degree greater than 1 of  $S_n$ . The immanant  $d_\chi : M_n \rightarrow \mathbb{C}$  associated with  $\chi$  is defined by

$$d_\chi(X) = \sum_{\sigma \in S_n} \chi(\sigma) \prod_{j=1}^n X_{j\sigma(j)}, \quad X = [X_{jk}] \in M_n.$$

Let  $\Omega_n$  be the set of all  $n$ -by- $n$  doubly stochastic matrices, that is, matrices with nonnegative real entries and each row and column sum is one. We say that a map  $T$  from  $\Omega_n$  into  $\Omega_n$

- is semilinear if  $T(\lambda S_1 + (1 - \lambda)S_2) = \lambda T(S_1) + (1 - \lambda)T(S_2)$  for all  $S_1, S_2 \in \Omega_n$  and for all real number  $\lambda$  such that  $0 \leq \lambda \leq 1$ ;
- preserves  $d_\chi$  if  $d_\chi(T(S)) = d_\chi(S)$  for all  $S \in \Omega_n$ .

We characterize the semilinear surjective maps  $T$  from  $\Omega_n$  into  $\Omega_n$  that preserve  $d_\chi$ , when the degree of  $\chi$  is greater than one.

**Key words.** Immanants, Linear preserver problems, Doubly stochastic matrices.

**AMS subject classifications.** 15A69, 15A60, 15A42, 15A45, 15A04, 47B49.

---

\*Received by the editors on December 10, 2015. Accepted for publication on February 22, 2017. Handling Editor: Raphael Lowey.

<sup>†</sup>CEAFEL and Faculdade de Ciências, Universidade de Lisboa, 1749-016 Lisboa, Portugal (mamonteiro@fc.ul.pt).

<sup>‡</sup>CMA and Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal (mrff@fct.unl.pt).