

## APPLICATION OF AN IDENTITY FOR SUBTREES WITH A GIVEN EIGENVALUE\*

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**Abstract.** For an Hermitian matrix whose graph is a tree and for a given eigenvalue having Parter vertices, the possibilities for the multiplicity are considered. If  $V = \{v_1, \dots, v_k\}$  is a fragmenting Parter set in a tree relative to the eigenvalue  $\lambda$ , and  $T_{i+1}$  is the component of  $T - \{v_1, v_2, \dots, v_i\}$  in which  $v_{i+1}$  lies, it is shown that  $\sum_i^k N_i = m_A(\lambda) + 2k - 1$ , in which  $N_i$  is the number of components of  $T_i - v_i$  in which  $\lambda$  is an eigenvalue. This identity is applied to make several observations, including about when a set of strong Parter vertices leaves only 3 components with  $\lambda$  and about multiplicities in binary trees. Furthermore, it is shown that one can construct an Hermitian matrix whose graph is a tree that has a strong Parter set  $V$  such that  $|V| = k$  for each  $k$  in  $1 \leq k \leq m - 1$  for given multiplicity  $m \geq 2$  of an eigenvalue  $\lambda$ . Finally, some examples are given, in which the notion of a fragmenting Parter set is used.

**Key words.** Tree, Eigenvalues, Hermitian matrices, Multiplicity, Parter vertex.

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