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# PERTURBATION RESULTS AND THE FORWARD ORDER LAW FOR THE MOORE-PENROSE INVERSE OF A PRODUCT*

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**Abstract.** New expressions are given for the Moore-Penrose inverse of a product $AB$ of two complex matrices. Furthermore, an expression for $(AB)^\dagger - B^\dagger A^\dagger$ for the case where $A$ or $B$ is of full rank is provided. Necessary and sufficient conditions for the forward order law for the Moore-Penrose inverse of a product to hold are established. The perturbation results presented in this paper are applied to characterize some mixed-typed reverse order laws for the Moore-Penrose inverse, as well as the reverse order law.

**Key words.** Moore-Penrose pseudo-inverse, Generalized inverses of a matrix product, Forward order law, Reverse order law.

**AMS subject classifications.** 15A09, 15A23, 15A24.

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