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THE HERMITIAN NULL-RANGE OF A MATRIX OVER A FINITE FIELD*

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Abstract. Let q be a prime power. For $u = (u_1, \dots, u_n), v = (v_1, \dots, v_n) \in \mathbb{F}_{q^2}^n$, let $\langle u, v\rangle := \sum_{i=1}^{n} u_i^q v_i$ be the Hermitian form of $\mathbb{F}_{q^2}^n$. Fix an $n \times n$ matrix M over $\mathbb{F}_{q^2}$. In this paper, it is considered the case $k = 0$ of the set $\mathrm{Num}_k(M) := \{\langle u, Mu\rangle \mid u \in \mathbb{F}_{q^2}^n, \langle u, u\rangle = k\}$. When M has coefficients in $\mathbb{F}_q$ the paper studies the set $\mathrm{Num}_k(M)_q := \{\langle u, Mu\rangle \mid u \in \mathbb{F}_q^n, \langle u, u\rangle = k\} \subseteq \mathbb{F}_q$. The set $\mathrm{Num}_1(M)$ is the numerical range of M, previously introduced in a paper by Coons, Jenkins, Knowles, Luke, and Rault (case q a prime $p \equiv 3 \pmod 4$), and by the author (arbitrary q). In this paper, it is studied in details $\mathrm{Num}_0(M)$ and $\mathrm{Num}_k(M)_q$ when $n = 2$. If q is even, $\mathrm{Num}_0(M)_q$ is easily described for arbitrary n. If q is odd, then either $\mathrm{Num}_0(M)_q = \{0\}$, or $\mathrm{Num}_0(M)_q = \mathbb{F}_q$, or $\sharp(\mathrm{Num}_0(M)_q) = (q + 1)/2$.

Key words. Numerical range, Finite field, Hermitian variety over a finite field.

AMS subject classifications. 12E20, 15A33, 15A60.

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