



## THE HAFNIAN AND A COMMUTATIVE ANALOGUE OF THE GRASSMANN ALGEBRA \*

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**Abstract.** A close relationship between the determinant, the pfaffian, and the Grassmann algebra is well-known. In this paper, a similar relation between the permanent, the hafnian, and a commutative analogue of the Grassmann algebra is described. Using the latter, some new properties of the hafnian are proved.

**Key words.** Hafnian, Permanent, Pfaffian, Determinant, Grassmann algebra.

**AMS subject classifications.** 15A15, 15A75, 15A78.

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