



ON THE LARGEST DISTANCE (SIGNLESS LAPLACIAN) EIGENVALUE OF NON-TRANSMISSION-REGULAR GRAPHS*

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Abstract. Let $G = (V(G), E(G))$ be a connected graph with n vertices and m edges. Let $D(G)$ be the distance matrix and $\lambda_1(D)$ be the distance spectral radius of G , respectively. The transmission $\text{Tr}(v_i)$ of $v_i \in V(G)$ is the sum of distances from v_i to all other vertices of G , i.e., the row sum D_i of $D(G)$ indexed by vertex v_i . Let $\text{Tr}(G)$ be the $n \times n$ diagonal matrix whose (i, i) -entry is equal to $\text{Tr}(v_i)$. The distance signless Laplacian matrix of G is defined as $D^Q(G) = \text{Tr}(G) + D(G)$ and its spectral radius is denoted by $\rho_1(D^Q)$. A connected graph G is t -transmission-regular if $\text{Tr}(v_i) = t$ for every vertex $v_i \in V(G)$; otherwise, G is non-transmission-regular. Suppose D_1 is the maximum row sum of $D(G)$. In this paper, $D_1 - \lambda_1(D)$ and $2D_1 - \rho_1(D^Q)$ are estimated in different ways for a k -connected non-transmission-regular graph. These obtained results are compared, and it is conjectured that $D_1 - \lambda_1(D) > \frac{1}{n+1}$. Moreover, it is shown that the conjecture holds for trees.

Key words. Distance (signless Laplacian) spectral radius, Maximum row sum, Connectivity, Non-transmission-regular graph.

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