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NORM INEQUALITIES RELATED TO CLARKSON INEQUALITIES*

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Abstract. Let A and B be $n \times n$ matrices. It is shown that if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both $A + B$, $A - B$ are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \leq 2^{p-1} \left(\|A\|_p^p + \|B\|_p^p \right) - \left(2^{p/2} - 2 \right) \left| \|A\|_p - \|B\|_p \right|^p,$$

and if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both A, B are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \geq 2 \left(\|A\|_p^p + \|B\|_p^p \right) + (2^{1-p/2} - 2^{2-p}) \left| \|A + B\|_p - \|A - B\|_p \right|^p.$$

These inequalities are reversed if $p = 2$, $1 \leq p \leq \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both $A + B$, $A - B$ are positive semidefinite, and if $p = 2$, $1 \leq p \leq \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both A, B are positive semidefinite, respectively. Commutative (or L_p) versions of these inequalities are also considered.

Key words. Clarkson inequalities, Hanner's inequality, Schatten p-norm, L_p function, Singular value.

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