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# EXPLICIT BLOCK-STRUCTURES FOR BLOCK-SYMMETRIC FIEDLER-LIKE PENCILS*

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**Abstract.** In the last decade, there has been a continued effort to produce families of strong linearizations of a matrix polynomial $P(\lambda)$, regular and singular, with good properties, such as, being companion forms, allowing the recovery of eigenvectors of a regular $P(\lambda)$ in an easy way, allowing the computation of the minimal indices of a singular $P(\lambda)$ in an easy way, etc. As a consequence of this research, families such as the family of Fiedler pencils, the family of generalized Fiedler pencils (GFP), the family of Fiedler pencils with repetition, and the family of generalized Fiedler pencils with repetition (GFPR) were constructed. In particular, one of the goals was to find in these families structured linearizations of structured matrix polynomials. For example, if a matrix polynomial $P(\lambda)$ is symmetric (Hermitian), it is convenient to use linearizations of $P(\lambda)$ that are also symmetric (Hermitian). Both the family of GFP and the family of GFPR contain block-symmetric linearizations of $P(\lambda)$, which are symmetric (Hermitian) when $P(\lambda)$ is. Now the objective is to determine which of those structured linearizations have the best numerical properties. The main obstacle for this study is the fact that these pencils are defined implicitly as products of so-called elementary matrices. Recent papers in the literature had as a goal to provide an explicit block-structure for the pencils belonging to the family of Fiedler pencils and any of its further generalizations to solve this problem. In particular, it was shown that all GFP and GFPR, after permuting some block-rows and block-columns, belong to the family of extended block Kronecker pencils, which are defined explicitly in terms of their block-structure. Unfortunately, those permutations that transform a GFP or a GFPR into an extended block Kronecker pencil do not preserve the block-symmetric structure. Thus, in this paper, the family of block-minimal bases pencils, which is closely related to the family of extended block Kronecker pencils, and whose pencils are also defined in terms of their block-structure, is considered as a source of canonical forms for block-symmetric pencils. More precisely, four families of block-symmetric pencils which, under some generic nonsingularity conditions are block minimal bases pencils and strong linearizations of a matrix polynomial, are presented. It is shown that the block-symmetric GFP and GFPR, after some row and column permutations, belong to the union of these four families. Furthermore, it is shown that, when $P(\lambda)$ is a complex matrix polynomial, any block-symmetric GFP and GFPR is permutationally congruent to a pencil in some of these four families. Hence, these four families of pencils provide an alternative but explicit approach to the block-symmetric Fiedler-like pencils existing in the literature.

**Key words.** Fiedler pencil, Block-symmetric generalized Fiedler pencil, Block-symmetric generalized Fiedler pencil with repetition, Matrix polynomial, Strong linearization, Symmetric strong linearization, Block Kronecker pencil, Extended block Kronecker pencil, Block minimal bases pencil.

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