



ON PROJECTION OF A POSITIVE DEFINITE MATRIX ON A CONE OF NONNEGATIVE DEFINITE TOEPLITZ MATRICES*

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Abstract. We consider approximation of a given positive definite matrix by nonnegative definite banded Toeplitz matrices. We show that the projection on linear space of Toeplitz matrices does not always preserve nonnegative definiteness. Therefore we characterize a convex cone of nonnegative definite banded Toeplitz matrices which depends on the matrix dimensions, and we show that the condition of positive definiteness given by Parter [*Numer. Math.* 4, 293–295, 1962] characterizes the asymptotic cone. In this paper we give methodology and numerical algorithm of the projection basing on the properties of a cone of nonnegative definite Toeplitz matrices.

This problem can be applied in statistics, for example in the estimation of unknown covariance structures under the multi-level multivariate models, where positive definiteness is required. We conduct simulation studies to compare statistical properties of the estimators obtained by projection on the cone with a given matrix dimension and on the asymptotic cone.

Key words. Banded Toeplitz matrix, Projection, Convex cone, Estimation, Covariance structure.

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