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POSITIVE AND Z-OPERATORS ON CLOSED CONVEX CONES*

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Abstract. Let K be a closed convex cone with dual K^* in a finite-dimensional real Hilbert space. A *positive operator* on K is a linear operator L such that $L(K) \subseteq K$. Positive operators generalize the nonnegative matrices and are essential to the Perron-Frobenius theory. It is said that L is a ***Z**-operator* on K if

$$\langle L(x), s \rangle \leq 0 \text{ for all } (x, s) \in K \times K^* \text{ such that } \langle x, s \rangle = 0.$$

The **Z**-operators are generalizations of **Z**-matrices (whose off-diagonal elements are nonpositive) and they arise in dynamical systems, economics, game theory, and elsewhere. In this paper, the positive and **Z**-operators are connected. This extends the work of Schneider, Vidyasagar, and Tam on proper cones, and reveals some interesting similarities between the two families.

Key words. Positive operator, Nonnegative matrix, Z-operator, Z-matrix, Lyapunov-like, Exponentially-positive.

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