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LINEAR PRESERVERS OF HADAMARD MAJORIZATION*

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Abstract. Let $\mathbf{M}_n$ be the set of all $n \times n$ real matrices. A matrix $D = [d_{ij}] \in \mathbf{M}_n$ with nonnegative entries is called doubly stochastic if $\sum_{k=1}^n d_{ik} = \sum_{k=1}^n d_{kj} = 1$ for all $1 \leq i, j \leq n$. For $X, Y \in \mathbf{M}_n$, it is said that X is Hadamard-majorized by Y, denoted by $X \prec_H Y$, if there exists an $n \times n$ doubly stochastic matrix D such that $X = D \circ Y$. In this paper, some properties of $\prec_H$ on $\mathbf{M}_n$ are first obtained, and then, the (strong) linear preservers of $\prec_H$ on $\mathbf{M}_n$ are characterized. For $n \geq 3$, it is shown that the strong linear preservers of Hadamard majorization on $\mathbf{M}_n$ are precisely the invertible linear maps on $\mathbf{M}_n$ which preserve the set of matrices of term rank 1. An interesting graph theoretic connection to the linear preservers of Hadamard majorization is exhibited. A number of examples are also provided in the paper.

Key words. Linear preserver, Hadamard majorization, Doubly stochastic matrix.

AMS subject classifications. 15A04, 15A21.

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